Gravitational Waves from Braneworld Inflation

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Talk Outline:

Braneworld scenario:

- Warped geometry
- Bulk scalar fields

Gravitational perturbations:

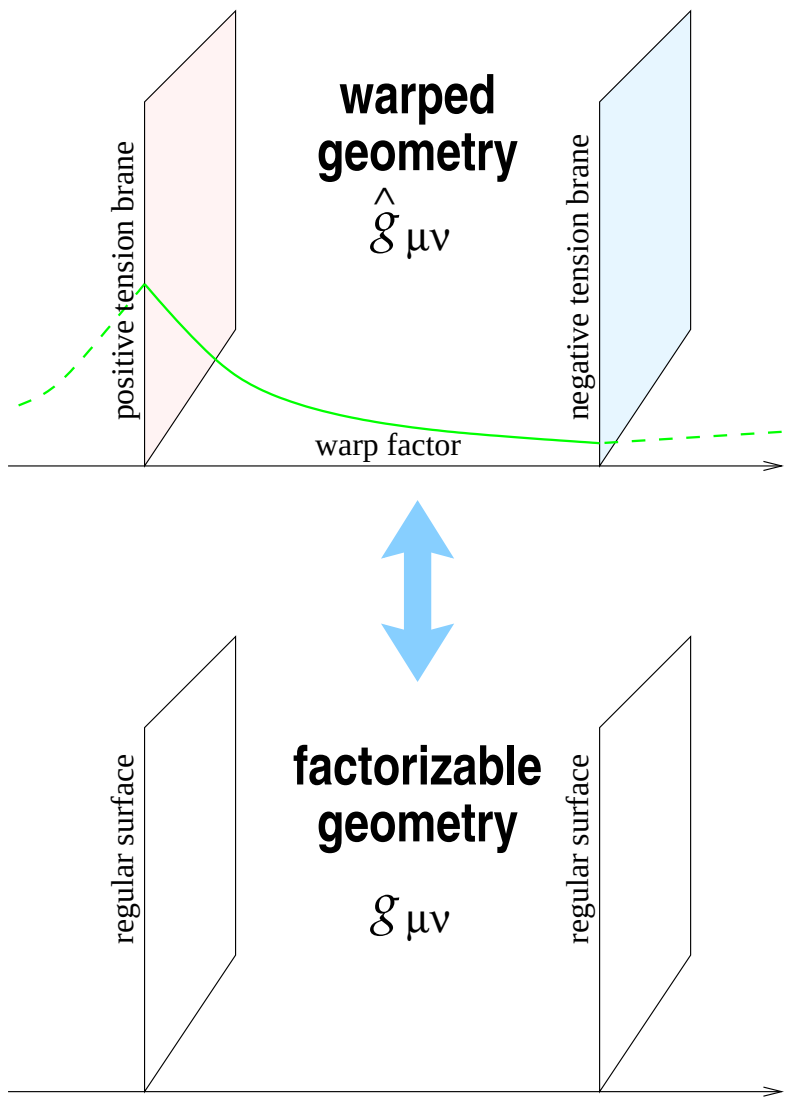
- Wave equation in conformal formalism
- Separation of extra dimension dependence
- Zero mode and Kaluza-Klein modes
- Effective potential and mass gap
- Amplitude of gravitational perturbations

Braneworld Geometry

➠ "Warped" braneworld metric:

$$\left[\begin{array}{c} \Omega = A^{-1} \\ d\chi = \frac{dy}{A(y)} \end{array}\right]$$

$$d\hat{s}^2 = dy^2 + A^2(y)\, ds_4^2 = \Omega^{-2}(\chi)\left(d\chi^2 + ds_4^2\right)$$



$$ds^2 = \Omega^2 d\hat{s}^2 = d\chi^2 + ds_4^2$$

Conformal transformation simplifies geometry!

Scalar Field Action

- Einstein-Hilbert action with scalar field:

$$S = \frac{1}{16\pi\kappa_D^2}\int \left\{R - (\nabla\phi)^2 - 2V(\phi)\right\}\sqrt{-g}\,d^Dx$$
$$-\frac{1}{8\pi\kappa_D^2}\sum_i \int \left\{[\mathcal{K}] + U(\phi)\right\}\sqrt{-h}\,d^{D-1}x$$

- Bulk Einstein and scalar field equations:

$$R_{\mu\nu} - \frac{1}{2}R\,g_{\mu\nu} = T_{\mu\nu}, \quad \Box\phi = \frac{\partial V}{\partial\phi}$$

- Bulk scalar field stress-energy tensor:

$$T_{\mu\nu} = \phi_{,\mu}\phi_{,\nu} + \left\{-\frac{1}{2}(\nabla\phi)^2 - V(\phi)\right\}g_{\mu\nu}$$

- Induced metric and extrinsic curvature:

$$g_{ab} = e^\mu_{(a)}e^\nu_{(b)}g_{\mu\nu}, \quad \mathcal{K}_{ab} = e^\mu_{(a)}e^\nu_{(b)}\nabla_\mu n_\nu$$

- Junction conditions at the branes:

$$[\mathcal{K}_{ab} - \mathcal{K}g_{ab}] = U(\phi)g_{ab}, \quad [n\cdot\nabla\phi] = \frac{\partial U}{\partial\phi}$$

Gravitational Perturbations

- Conformally-factorizable metric and perturbations:

$$d\hat{s}^2 = \Omega^{-2}(\chi)\left(d\chi^2 + g_{ab}(x^i)dx^a dx^b\right), \qquad \delta\hat{g}_{\mu\nu} = \Omega^{-2}h_{\mu\nu}$$

- Linearized Einstein equations: $\delta\hat{R}^{\mu}{}_{\nu} = 0$

$$\Box h_{\mu\nu} - 2\nabla^{\rho}\nabla_{(\mu}h_{\nu)\rho} - (N-2)\Omega^{-1}\Omega^{,\rho}\nabla_{\rho}h_{\mu\nu} + 2R^{\rho}{}_{(\mu}h_{\nu)\rho} = 0$$

$$\nabla^{\mu}\Omega\, h_{\mu\nu} = 0, \quad \nabla^{\mu}h_{\mu\nu} = 0, \quad h = 0 \quad \text{(gauge choice)}$$

- Dependence on extra dimension separates:

$$h_{ab} = \psi(\chi)H_{ab}(x^i), \qquad (\nabla^a H_{ab} = 0, \;\; H = 0)$$

- The gravitational wave problem reduces to:

$$\psi'' - (N-2)\Omega^{-1}\Omega'\psi' = -m^2\psi$$

$$\Box H_{ab} - 2\nabla^c\nabla_{(a}H_{b)c} + 2(N-2)KH_{ab} = m^2 H_{ab}$$

- Perturbation spectrum:

Zero mode:	*KK modes:*
$\psi(\chi) = \text{const}$	$\psi'\vert_{\text{brane}} = 0$

- Zero mode normalization: $M_{(N-1)}^{N-3} = M_{(N)}^{N-2}\ell_{\text{eff}}$

$$\frac{H}{M_4(\text{inf})} = \frac{H}{M_4(\text{now})}\left[\frac{\ell_{\text{eff}}(\text{now})}{\ell_{\text{eff}}(\text{inf})}\right]^{\frac{1}{2}}$$

Perturbation Spectrum

- Self-adjoint EV problem: $\Psi = \Omega^{-\frac{N-2}{2}}\psi, \quad \bar{\Psi} = \mathcal{D}_+\Psi$

$$-\mathcal{D}_-\mathcal{D}_+\Psi \equiv -\Psi'' + V_{\text{eff}}\,\Psi = m^2\Psi, \qquad \mathcal{D}_+\Psi|_{\text{brane}} = 0$$

$$-\mathcal{D}_+\mathcal{D}_-\bar{\Psi} \equiv -\bar{\Psi}'' + \bar{V}_{\text{eff}}\,\bar{\Psi} = m^2\bar{\Psi}, \qquad \bar{\Psi}|_{\text{brane}} = 0$$

$$\mathcal{D}_\pm = \partial_\chi \pm p, \quad \{V, \bar{V}\}_{\text{eff}} = \mp p' + p^2, \quad p = \frac{N-2}{2}\,(\ln\Omega)'$$

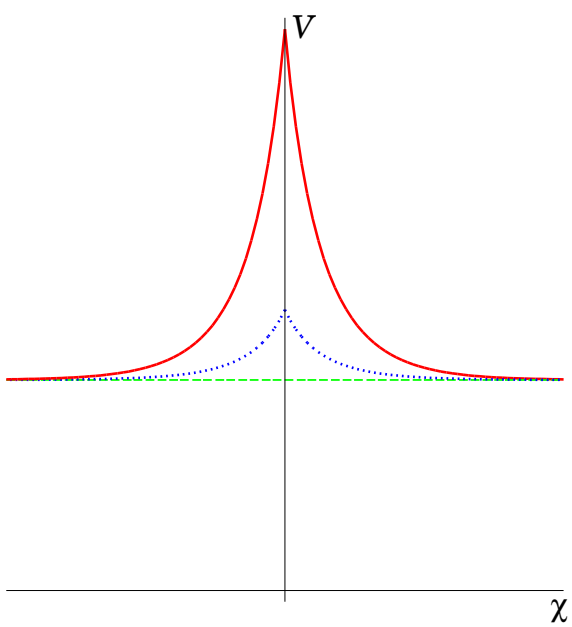
- Effective potential bound for negative curvature bulk:

$$V_{\text{eff}} = \frac{1}{4}\frac{N-2}{N-1}\left[R - \Omega^{-2}\hat{R}\right] = \left(\frac{N-2}{2}\right)^2 K - \frac{1}{4}\frac{N-2}{N-1}\frac{\hat{R}}{\Omega^2}$$

- Effective potential bound for scalar field bulk:

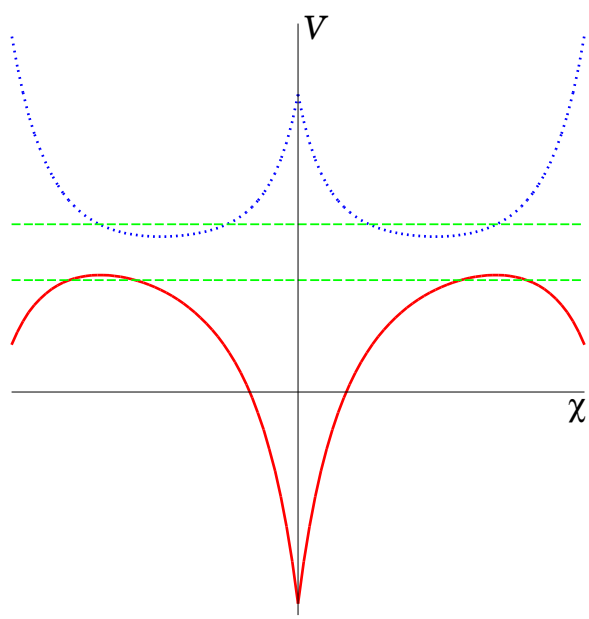
$$\bar{V}_{\text{eff}} = \frac{N-2}{2}\,K + \frac{1}{2}\,\phi'^2 + \frac{(N-4)(N-2)}{4}\,[(\ln\Omega)']^2$$

AdS bulk



$$m \geq \frac{N-2}{2}\,H$$

scalar field bulk



$$m \geq \sqrt{\frac{N-2}{2}}\,H$$

Gravitational Wave Amplitude

- Dimensional reduction of Einstein action:

$$S = M_S^3 \int R[g_{\mu\nu}]\sqrt{-g}\, d^5x = M_{P,\text{eff}}^2 \int R[g_{ab}]\sqrt{-g}\, d^4x$$

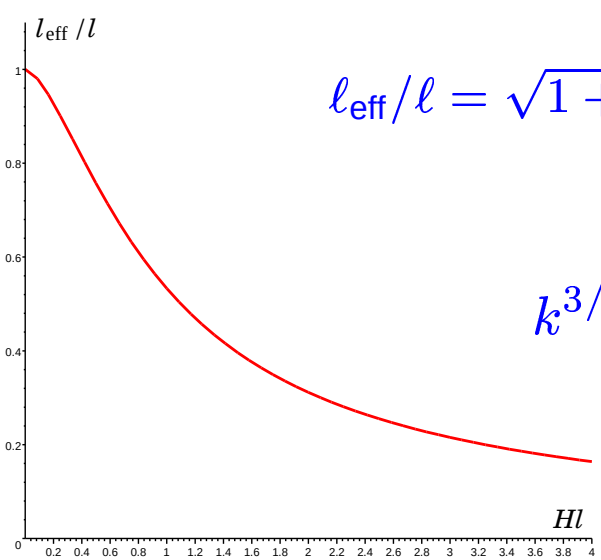
- 4D Planck mass is an effective quantity:

$$M_{P,\text{eff}}^2 = M_S^3\, \ell_{\text{eff}}, \quad \ell_{\text{eff}} = \int_{\chi_1}^{\chi_2} \Omega^{-3}(\chi)\, d\chi = \int_{y_1}^{y_2} A^2(y)\, dy$$

- Amplitude of gravitational perturbations:

$$k^{3/2} h_k = \frac{H}{M_{P,\text{inf}}} = \frac{H}{M_{P,\text{now}}} \sqrt{\frac{\ell_{\text{now}}}{\ell_{\text{inf}}}}$$

- Randall-Sundrum model: single brane in AdS bulk



$$\ell_{\text{eff}}/\ell = \sqrt{1 + H^2\ell^2} + H^2\ell^2 \ln\left(\frac{\sqrt{1+H^2\ell^2}-1}{H\ell}\right)$$

$$k^{3/2} h_k = \frac{H}{M_{P,\text{eff}}} = \frac{H}{M_P}\sqrt{\frac{3}{2}H\ell}$$

$$(H\ell \gg 1)$$

Summary

- Gravitational waves in braneworld scenario:
  - Vacuum fluctuations are generated by inflation
  - Gravitational waves sensitive only to geometry
  - Extra dimension dependence separates
  - Zero mode has 2 DOF, KK modes have 5 DOF
  - Spectrum of Kaluza-Klein modes can be analyzed
- Massive graviton modes:
  - Mass gap in KK spectrum $m \geq \sqrt{3/2}\, H$
  - Massive KK graviton modes are not generated!
- Zero graviton mode:
  - No massless (gravi)scalar and (gravi)vector from bulk graviton projection
  - Gravitational perturbations amplitude is $H/M_{P,\text{eff}}$
  - May be different from 4D prediction!
  - Expect T/S from braneworld inflation to be model-dependent