Neutrino Mixing in the Early Universe

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ν oscillations and Big Bang Nucleosynthesis

Q: What impact do neutrino oscillations have upon our understanding of BBN light element abundances?

A: It all depends on the size of the relic neutrino asymmetries.
i.e. whether the neutrino ensembles have chemical potentials.

Our knowledge of the neutrino mixing matrix allows us to set a tight constraint on the energy density in relic neutrinos.

Review of BBN issues for active-sterile mixing

Most attention on neutrino oscillations at the time of BBN has focused on active-sterile neutrino mixing. Enqvist, Kainulainen, Thomson, Foot, Volkas, Shi, Fuller, Abazajian, Bell, Wong, di Bari, Dolgov, etc...

Even if we start out with no sterile neutrino around, oscillations can populate the sterile neutrinos before BBN. $\nu_{\mathrm{active}} \leftrightarrow \nu_{\mathrm{sterile}}$

This is bad because:

The universe would expand faster
Weak interaction rates would freeze out earlier
Larger neutron to proton ratio and hence more Helium

BBN says N < 4 → Sterile neutrinos are cosmologically disfavoured if they mix significantly with active neutrinos.

ALL “3+1” and “2+2” models which accommodate LSND are problematic for BBN.

See Abazajian (2002); di Bari (2001), for recent analyses.

HOWEVER… there are ways out…

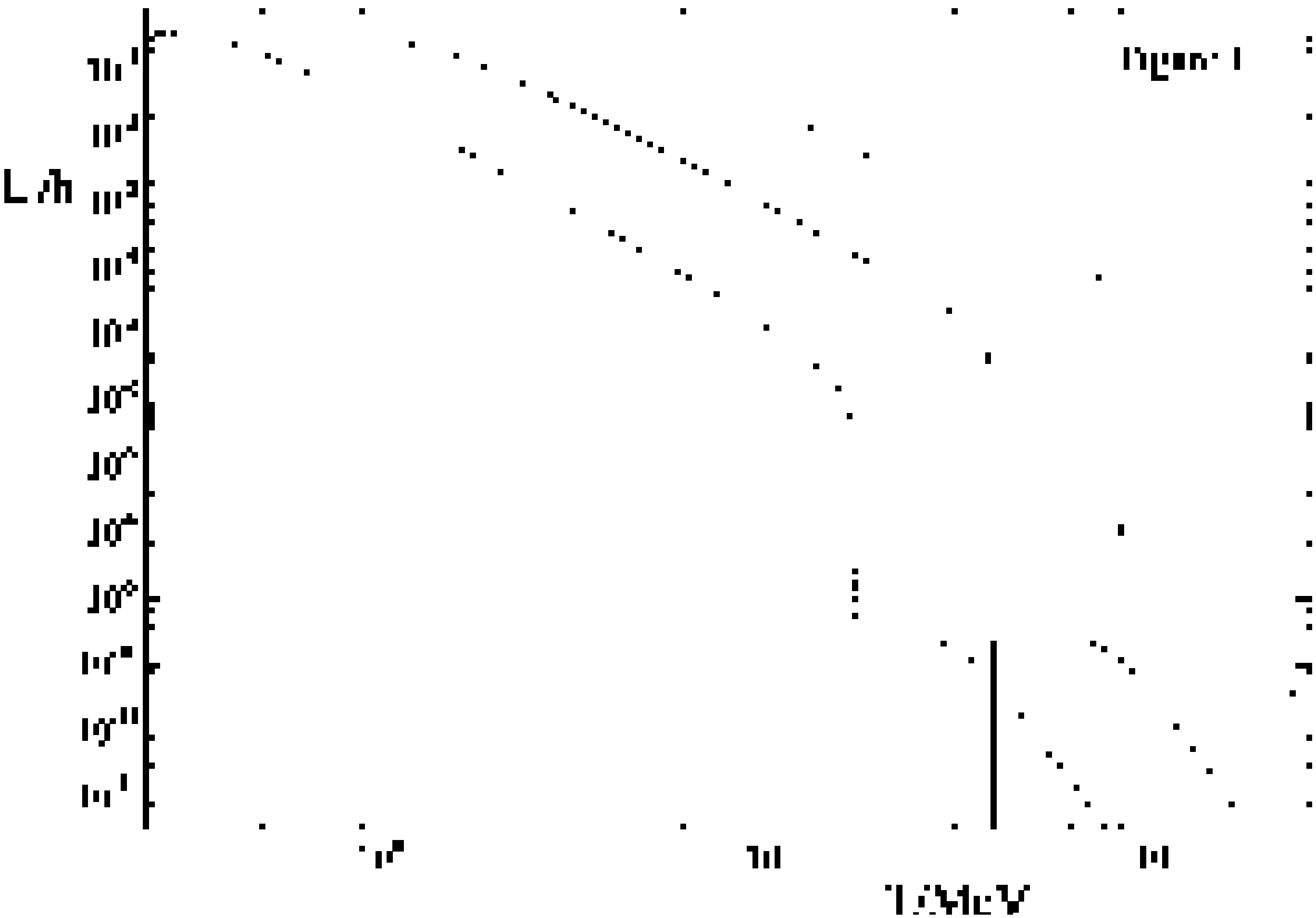
- Equilibration of the sterile is avoided if a lepton asymmetry is present
→the mixing angle is suppressed due to the refractive index

Foot & Volkas (1995)

- Scenarios with a low reheating temperature
→ Next talk - K. Kohri

Active-sterile oscillation modes can generate large neutrino asymmetries.

Foot, Thomson & Volkas (1995)



Active-Active Oscillations

Oscillations between active neutrino species have received much less attention than active sterile oscillations because:

1. Oscillations do nothing if we have equal numbers of each flavour
But there may very well be asymmetries between the flavours.

2. Its a much harder problem.
Neutrino-neutrino forward scattering makes things highly non-linear.

Lepton Asymmetries

Baryon asymmetry: $$B = \frac{n_b - n_{\bar{b}}}{n_\gamma} \sim 10^{-10}$$

Lepton asymmetry: → only very weak constraints.

Charge neutrality of the universe prevents a large asymmetry in the charged leptons, but a large lepton number could reside in the neutrino sector
-- and we have only indirect information about the relic neutrinos.

In thermal equilibrium, the neutrinos will have Fermi-Dirac distributions:

$$f(p,\xi) = \frac{1}{1+\exp(p/T-\xi)} \qquad \xi = \text{chemical potential}$$

Lepton asymmetries:

$$L_\alpha = \frac{n_{\nu_\alpha} - n_{\bar{\nu}_\alpha}}{n_\gamma} \approx \frac{\pi^2}{12\varsigma(3)}\xi_\alpha$$

Such degeneracies increase the effective number of species in equilibrium:

$$\Delta N_\nu = \frac{30}{7}\left(\frac{\xi}{\pi}\right)^2 + \frac{15}{7}\left(\frac{\xi}{\pi}\right)^4$$

ν_e directly affects neutron-proton equilibrium:

$$n + \nu_e \leftrightarrow p + e \qquad n/p \approx \exp\left[-(m_n - m_p)/T - \xi_e\right]$$

What do we know about the relic neutrinos?

Can't detect them directly. (E=1$\times$10-4 eV today)

BBN sets weak bounds on the asymmetries:

$$\text{Very weak bound for } \nu_{\mu,\tau}: \quad \left|\xi_{\mu,\tau}\right| < 2.6$$

$$\text{Stronger bound for } \nu_e: \quad -0.01 < \xi_e < 0.22$$

Hansen et al. (2001)

LMA Solar Neutrino Solution

The Large Mixing Angle solution looks more and more like the correct resolution of the solar neutrino anomaly → soon to be checked by KamLaND

Best fit mixing parameters:

$$\delta m_0^2 \approx 4\times 10^{-5}\ \mathrm{eV}^2$$
$$\sin^2 2\theta_0 \approx 0.8$$

Large-angle mixing could potentially equilibrate the flavours.

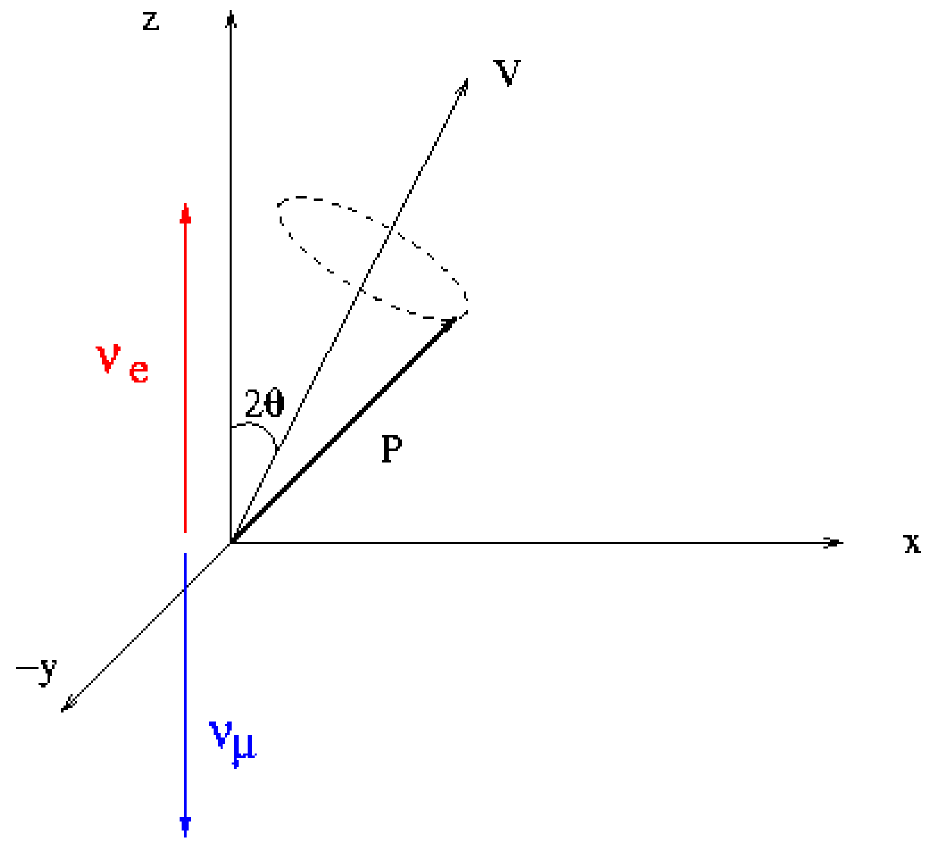
Lunardini and Smirnov (2001).

Matter effects are quite significant and must be included to determine if equilibration would take place before BBN.

Dolgov et al. (2002), Abazajian, Beacom and Bell (2002), Wong (2002).

Density matrix parameterization:

$$\rho = \begin{pmatrix} \rho_{ee} & \rho_{e\mu} \\ \rho_{\mu e} & \rho_{\mu\mu} \end{pmatrix} = \frac{1}{2}\left[P_0 + \boldsymbol{\sigma} \bullet \mathbf{P}\right]$$



"Polarisation vector"

$$\mathbf{P} = \left(P_x, P_y, P_z\right)$$

Oscillations are described by the precession of the P vector
-- like a spin precessing in a magnetic field.

Initial condition: $$\mathbf{P} \propto \left[f_e(\xi_e) - f_\mu(\xi_\mu)\right]\hat{z}$$

Evolution equations:

$$\partial_t \mathbf{P}_p = \left(\mathbf{A}_p + \alpha\,\mathbf{I}\right) \times \mathbf{P}_p$$
$$\partial_t \overline{\mathbf{P}}_p = \left(-\mathbf{A}_p + \alpha\,\mathbf{I}\right) \times \overline{\mathbf{P}}_p$$

A=vacuum mixing term + non-neutrino background

I= neutrino-neutrino forward scattering term

$$\mathbf{A}_p = \frac{\delta m_0^2}{2p}\left(\sin 2\theta_0 \hat{x} - \cos 2\theta_0 \hat{z}\right) + V_B \hat{z}$$

$$\mathbf{I} = \int \frac{d^3(p/T)}{(2\pi)^3}\left[\mathbf{P}_p - \overline{\mathbf{P}}_p\right]$$

Behaviour of single mode in absence of ν–ν forward scattering term:

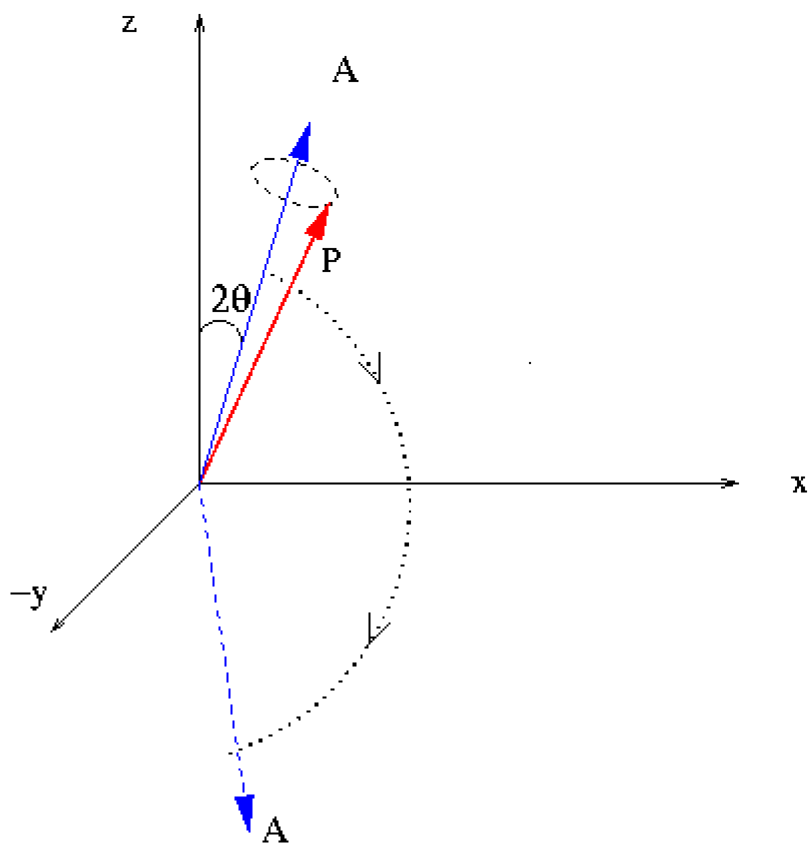
$$\partial_t \mathbf{P}_p \approx \mathbf{A}_p \times \mathbf{P}_p$$

$$\mathbf{A}_p = \frac{\delta m_0^2}{2p}\left(\sin 2\theta_0 \hat{x} - \cos 2\theta_0 \hat{z}\right) - \frac{8\sqrt{2} G_F p}{3 M_W^2} E_e \hat{z}$$

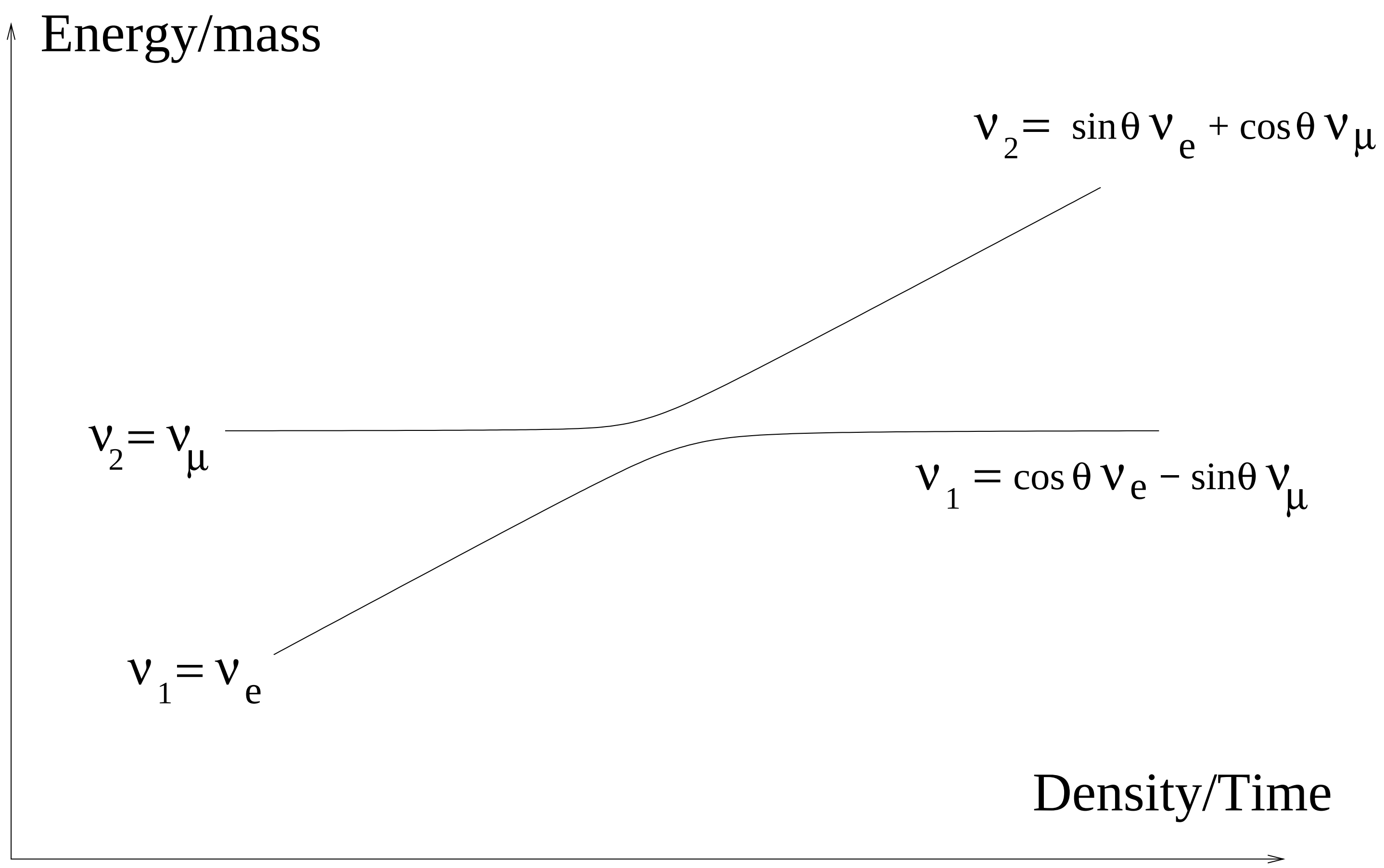
- The thermal potential is initially large and decreases slowly as the temperature falls

- A rotates from the Z-axis to a direction specified by the vacuum mixing parameters.

- The polarisation vector is initially aligned with A, and follows A as it makes this transition – this is just an adiabatic MSW transition


z
A
P
2θ
x
−y
A

MSW transitions



The neutrino-neutrino forward scattering term makes the problem highly non-linear.

Note that this term includes both diagonal and off-diagonal refractive indices, the off-diagonal contributions coming from forward scattering processes of the type:

$$\nu_\alpha(p)+\nu_\beta(k)\rightarrow\nu_\alpha(k)+\nu_\beta(p)$$

Pantaleone (1992).

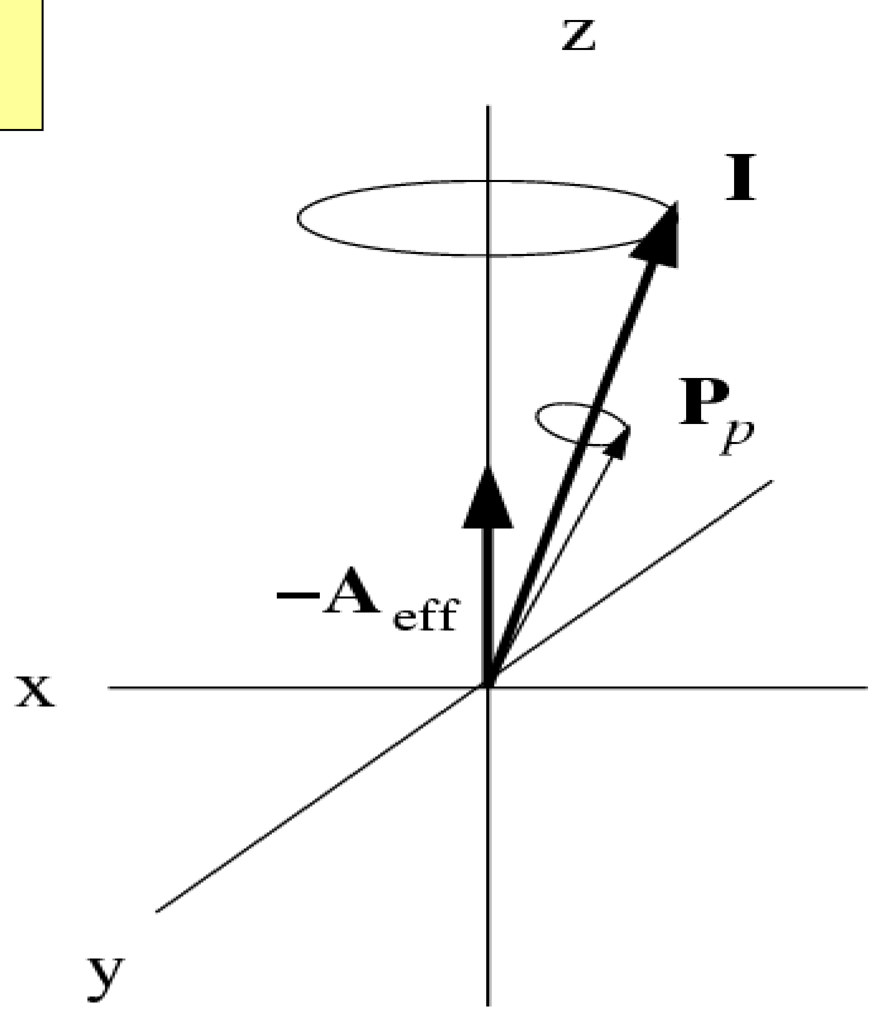
The non-linear term dominates in size as long as the initial asymmetry is larger than about:

$$L>10^{-5}$$

…that is, it dominates for all initial asymmetries of interest.

$$\partial_t \mathbf{P}_p = \left(\mathbf{A}_p + \alpha\,\mathbf{I}\right)\times \mathbf{P}_p$$
$$\partial_t \overline{\mathbf{P}}_p = \left(-\mathbf{A}_p + \alpha\,\mathbf{I}\right)\times \overline{\mathbf{P}}_p$$

$$\mathbf{I} = \int (\mathbf{P}_p - \overline{\mathbf{P}}_p)$$



Synchronisation

•In the absence of neutrino-neutrino forward scattering, each momentum mode has a different oscillation frequency.

•Including nu-nu forward scattering pins all the momentum modes together so they oscillate in sync. Samuel; Kostelecky & Samuel; Pantaleone.

The polarisation vector for each momentum mode is pinned to the collective polarisation vector I.

Pastor, Raffelt and Semikoz (2002)

The evolution of the collective polarisation is determined according to:

$$\partial_t \mathbf{I} \approx \mathbf{A}_{\mathrm{eff}} \times \mathbf{I}$$

$$\mathbf{A}_{\mathrm{eff}} \approx \frac{1}{\mathbf{I}^2} \int \mathbf{A}_p \left(\mathbf{P}_p + \mathbf{P}_p \right) \bullet \mathbf{I}$$

When the neutrino self potential dominates, it synchronises the ensemble so that all neutrinos behave as though they have the same effective momentum.

Parameters describing the evolution of the collective polarisation:

$$\mathbf{A}_{\text{eff}} \approx \Delta_{\text{sync}} \left(\sin 2\theta_{\text{sync}} \hat{x} - \cos 2\theta_{\text{sync}} \hat{z} \right)$$

$\Delta_{\text{sync}} \propto \xi$ Very sensitive to initial asymmetry.

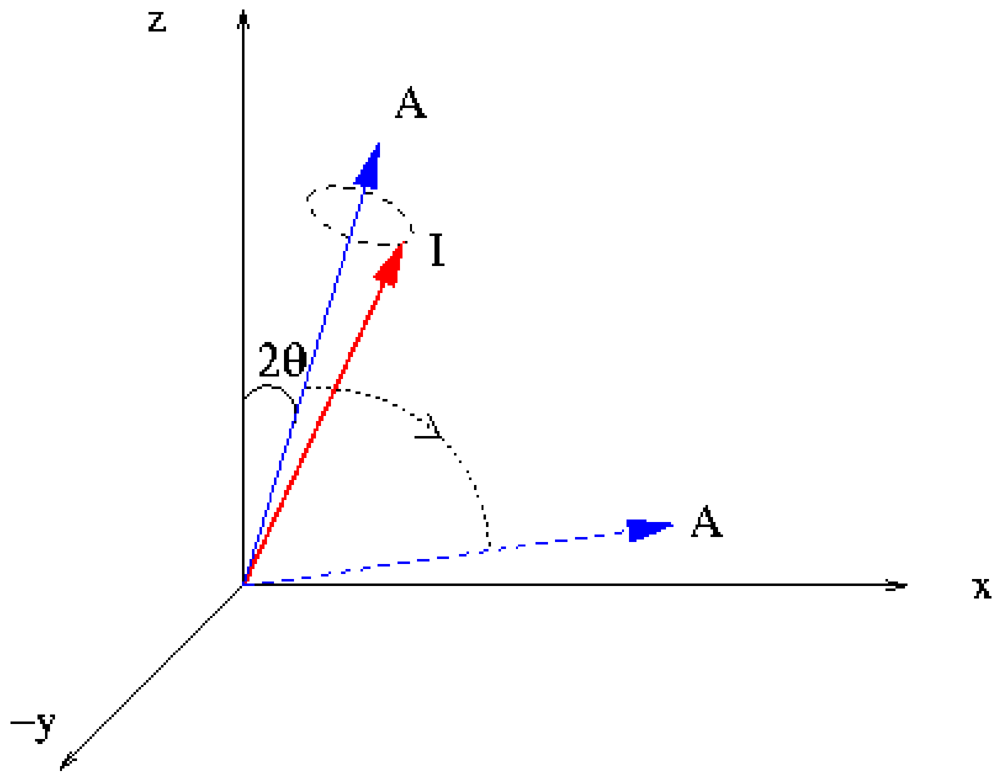
Effective mixing angle , however is insensitive to the initial asymm:

$$\sin^2 2\theta_{\text{sync}} = \frac{\sin^2 2\theta_0}{\sin^2 2\theta_0 + \left[\cos 2\theta_0 - V_B(p_{\text{sync}}) \right]^2}$$

$$\frac{p_{\text{sync}}}{T} = \pi \sqrt{1 + \xi^2 / 2\pi^2} \approx \pi$$

Abazajian, Beacom and Bell (2002);
Wong (2002).

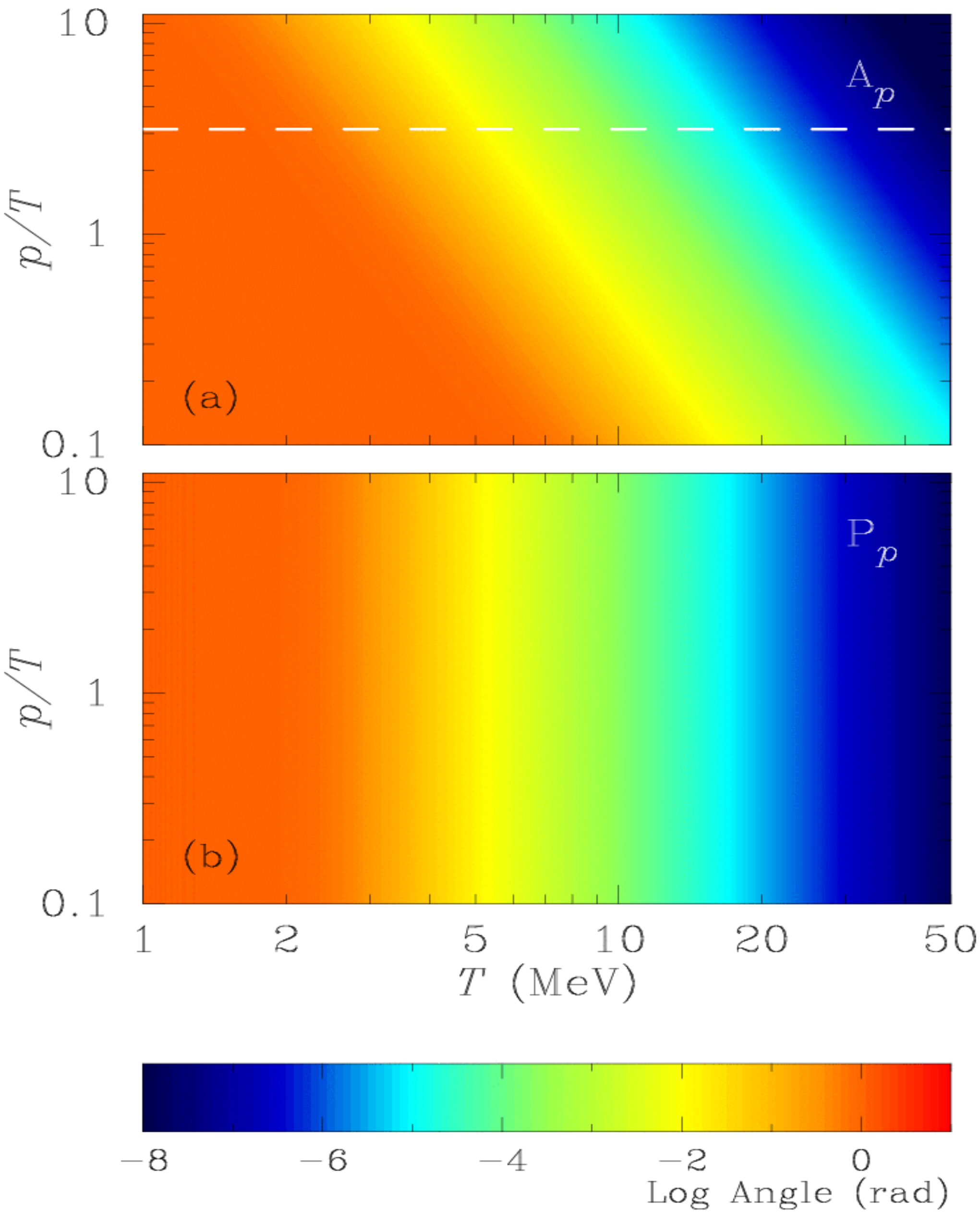
It is this “synchronised mixing angle” that determines when the flavour equilibration occurs.

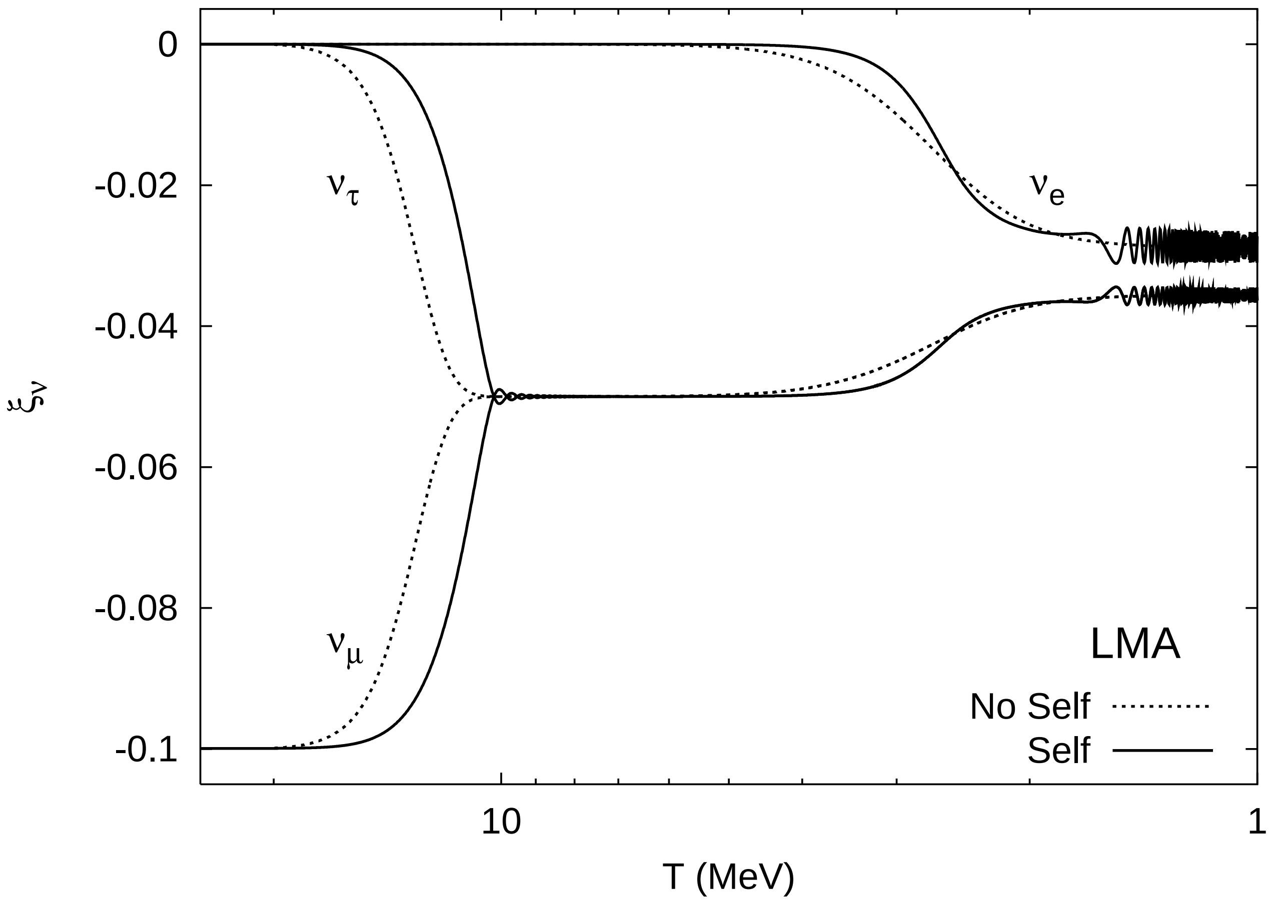


In the standard three-flavour picture of neutrino mixing:

$\nu_\mu - \nu_\tau$ equilibration takes place at T ~ 10 MeV

$\nu_e - \nu_\mu / \nu_\tau$ equilibration takes place at T ~ 2 MeV, just before BBN.


10
A_p
p/T
1
(a)
0.1
10
P_p
p/T
1
(b)
0.1
1
2
5
10
20
50
T (MeV)
−8
−6
−4
−2
0
Log Angle (rad)



Dolgov, Hansen, Pastor, Petcov, Raffelt &Semikoz. (2002)

New Constraints:

$$\xi_e^f \approx \left(\frac{1-\cos 2\theta_0}{2}\right)\xi_\mu^i$$

Using the best fit value of the LMA mixing angle $\sin^2 2\theta_0 \approx 0.8$

$$\xi_e^f < 0.04 \quad \Rightarrow \quad \xi_\mu^i < 0.3$$

Collisional processes will help make the equilibration more complete, as does non-zero U_{e3}.

Degenerate BBN is eliminated since chemical potentials in any flavour will effectively impact neutron-proton equilibrium.

Summary

- The LMA (large mixing angle) solar neutrino solution
  → equilibration of neutrino flavours just before BBN

- The equilibration takes place via a synchronised MSW transition

- The stringent constraints on ν_e apply to all three flavours

- If a non-standard contribution to the relativistic energy density were to be detected, say, via the CMB, its origin would be something more exotic than neutrino degeneracy.

- We have a tight limit on the relic neutrino number density