Testing Gravity Against the $z \sim 3$ Integrated Sachs-Wolfe Effect

Pengjie Zhang
Shanghai Astronomical Observatory

External correlations of the CMB and cosmology
May, 2006, Fermilab

The Integrated Sachs-Wolfe Effect

- Refer to all previous talks
- Basic concepts

- It measures the time variation of gravitational potential.

$$\frac{\Delta T}{T_{\mathrm{CMB}}} = \int [\dot{\phi} - \dot{\psi}] a d\chi$$

Detectability.

- It has been detected from the CMB-LSS cross correlation (Fosalba et al. 2003; Scranton et al. 2003; Afshordi et al. 2004; Boughn & Crittenden 2004; Fosalba & Gaztanaga 2004; Nolta et al. 2004; Vielva et al. 2004; Padmanabhan 2005).
- It will be measured using other LSS tracers such as lensing (Seljak & Zaldarriaga 1999).

- $\dot{\phi} = 0$ in a universe satisfying the following conditions
  - $\Omega_m = 1, \Omega_{\mathrm{DE}} = 0, \Omega_k = 0$
  - standard gravity (the General Relativity)
  - linear regime

- Its detection generally implies
  - $\Omega_m \neq 1, \Omega_{\mathrm{DE}} \neq 0$, given a flat prior
  - deviation from the general relativity at relevant scales. DGP, $f(R)$, etc.

Outine of the talk

For future surveys!

- Isolating the decay rate of gravitational potential combining the ISW measurement and lensing measurement.
  - Scale independence of $\dot{\phi}/\phi$ is required. Applicable to general relativity and some modified gravities such as DGP

- Testing gravity against $z \sim 3$ ISW effect.
  - Some modified gravity (e.g. exponential $f(R)$ gravity) has detectable ISW effect at $z \sim 3$.

Isolating the decay rate of gravitational potential

- Given a flat prior, detections of the gravitational potential decay have already provided strong evidence of the existence of dark energy (e.g. Corasaniti et al. (2005); Pogosian et al. (2005)).

- Key information in the ISW measurement is $\dot{\phi}/\phi$, which depends on $\Omega_m, \Omega_{\rm DE}, w$

- But the measured correlation depends on many nuisance parameters: $P_{\rm initial}(k), \sigma_8, H_0, b_g(k), r_g(k)$. Hard to predict from first principles

- Gravitational lensing measures the projected gravitational potential.
  - $\Phi(\hat{n}) = -\int d\chi(\phi - \psi)W(\chi, \chi_s)$.
  - $W = (1 - \chi/\chi_s)/\chi$
  - χ and χ_s are the comoving angular diameter distance to lens and source.

- The ISW effect-LSS and lensing-LSS probe the same 3D gravitational potential ϕ, with different pre-factors. For such narrow redshift bin,

  the ISW-galaxies cross correlation measures $\langle\dot{\phi}\delta_g\rangle$

  the lensing-galaxies cross correlation measures ϕ-δ_g

- The ratio of the two correlations in the same redshift bin at the same scale then measures $\dot{\phi}/\phi$ with pre-factor which only depends on the geometry of the Universe.

- Advantages:
  - Minimum parameter space. Does not depend on $b_g(k)$, $P_{\mathrm{initial}}(k)$, σ_8, etc.
  - Redshift information is recovered.
  - Minimum sample variance. The two cross correlation measurements are correlated, errors in the denominator and numerator partly cancel. The S/N of the measured ratio is slightly better than the ISW-LSS measurement.

More details

- ISW-galaxy cross correlation. Galaxies: $z_1 < z < z_2$.

$$
\begin{aligned}
C_{Ig} &= 4\pi \int \Delta^2_{\phi\delta}(k, z=0)\frac{dk}{k} A_I(k,l)A_g(k,l) \\
&\simeq \frac{4\pi^2}{l^3}\int_{\chi_1}^{\chi_2} \Delta^2_{\phi\delta}(\frac{l}{\chi}, z=0) n_g D_g \dot{D}_\phi a\chi d\chi \\
&\simeq \frac{4\pi^2}{l^3}[\Delta^2_{\phi\delta}(\frac{l}{\chi}, z=0) n_g D_g \dot{D}_\phi a\chi]|_{\bar{z}}(\chi_2 - \chi_1)
\end{aligned}
$$

Limber's approximation

Narrow z bin

- Lensing-galaxy cross correlation.

$$
C_{\Phi g} \simeq \frac{4\pi^2}{l^3}[\Delta^2_{\phi\delta}(\frac{l}{\chi}, z=0) n_g D_g D_\phi W\chi]|_{\bar{z}}(\chi_2 - \chi_1)
$$

- Advantages:
  - Minimum parameter space. Does not depend on $b_g(k)$, $P_{\text{initial}}(k)$, σ_8, etc.
  - Redshift information is recovered.
  - Minimum sample variance. The two cross correlation measurements are correlated, errors in the denominator and numerator partly cancel. The S/N of the measured ratio is slightly better than the ISW-LSS measurement.
  - Redshift information is recovered.

- $\frac{C_{Ig}}{C_{\Phi g}} \simeq f(\bar{z})$. $f(z) = [(d\ln D_\phi/d\ln a)]aH/W(\chi)$.
- Optimal estimator of $f(z)$
  - $\hat{f} = \sum_l C_{Ig}(l)w_l / \sum C_{\Phi g} w_l$
  - the minimum variance estimator $w_l = \frac{C_{Ig}(l)}{\Delta C^2_{Ig}(l)}$
  - the minimum variance (*in the limit that the sample variance of the ISW effect is dominant*) is

$$\frac{\Delta f^2}{f^2} \simeq \left(\sum_l \frac{C^2_{Ig}}{\Delta C^2_{Ig}}\right)^{-1} \simeq \left(\sum_l \frac{(2l+1)f_{\text{sky}}C^{\text{ISW}}}{r^2 C^{\text{CMB}}}\right)^{-1}$$

(nearly) Ideal surveys

- Constructing the projected gravitational potential Φ.
  - cosmic shear
  - CMB lensing (Seljak & Zaldarriaga 1999a; Zaldarriaga & Seljak 1999; Hu & Okamoto 2002)
  - 21cm background lensing (Zahn & Zaldarriaga 2005)
  - cosmic magnification (Zhang & Pen 2005)

- Galaxies. (1) with sufficiently high number density to high redshifts. (2) With Photo-z or spec-z. (3) Cover a large fraction of the sky
  - Optical surveys (e.g. LSST)
  - 21cm surveys (e.g. SKA). 10^9 galaxies can be detected at $z \sim 3$.

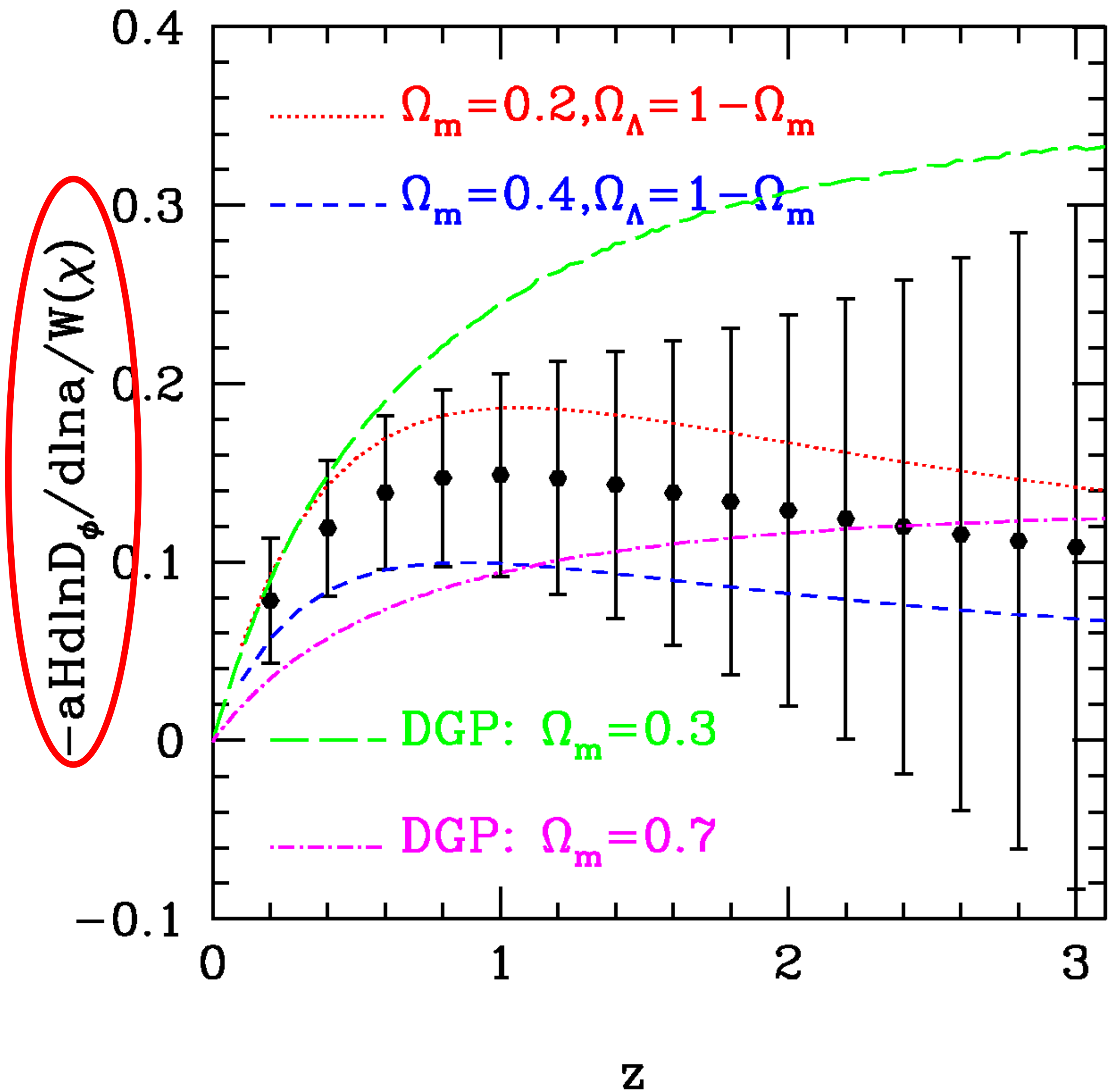


Fig. 1.— The accuracy of measured $f \equiv aHd\ln D_\phi/d\ln a/W(\chi)$, assuming both CMB and galaxy surveys cover the full sky. Fiducial cosmology has $(\Omega_m, \Omega_\Lambda, \Omega_b, h, \sigma_8) = (0.268, 0.732, 0.044, 0.71, 0.84)$. Transfer function is calculated using the fitting formula of (Eisenstein & Hu 1998).

f(z):sensitive to dark energy and the nature of gravity

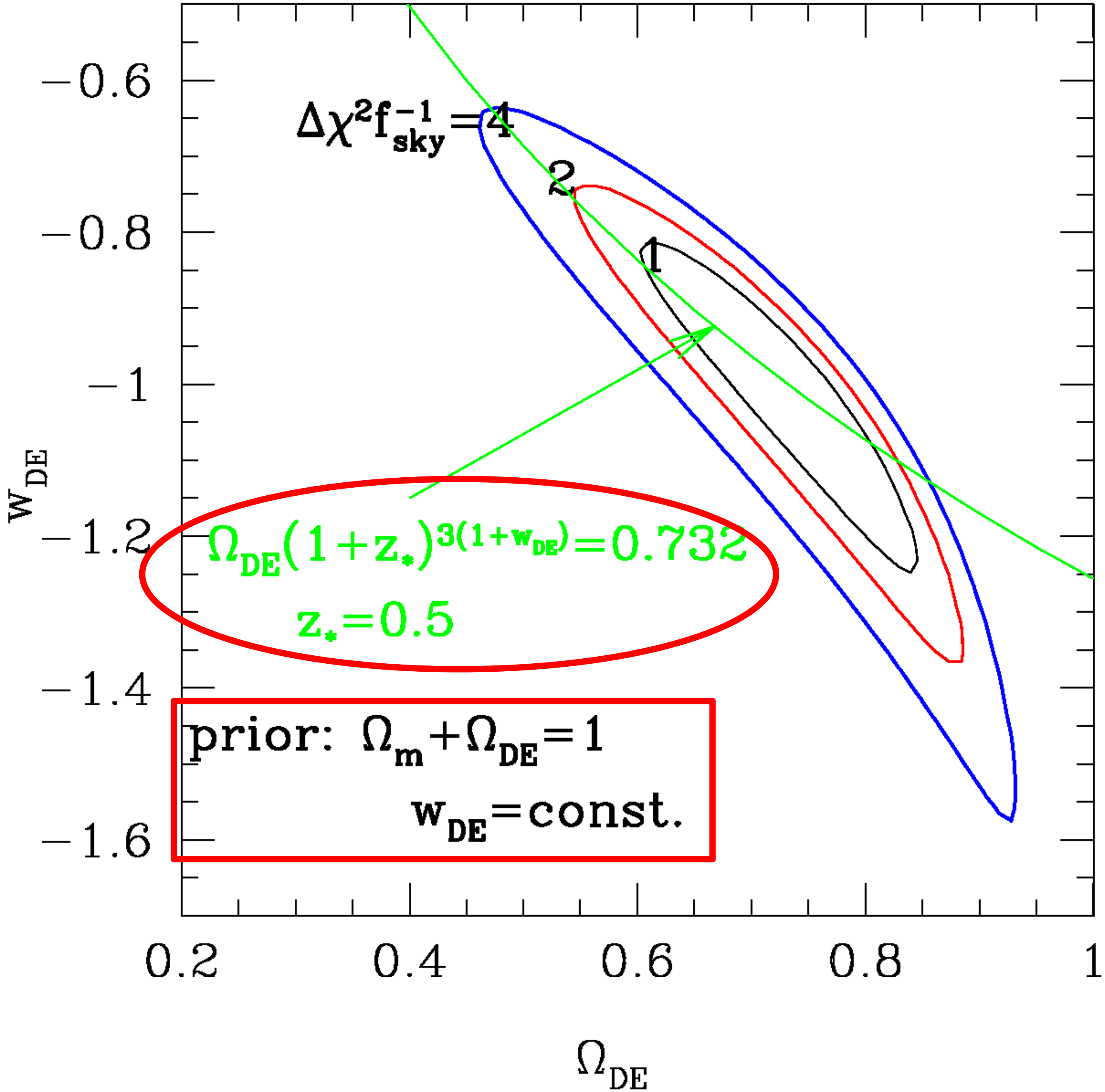


Fig. 2.— Ω_{DE}-w_{DE} contour. Since most signals come from $z_* \sim 0.5$, $d\ln D_\phi/d\ln a$ at $z_* \sim 0.5$ should be close to the fiducial cosmology in order to get a good fit. This requires that the dark energy density $\Omega_{DE}(1+z_*)^{3(1+w_{DE})} \simeq \Omega_\Lambda^{\text{fiducial}}$. We plot the line with $z_* = 0.5$.

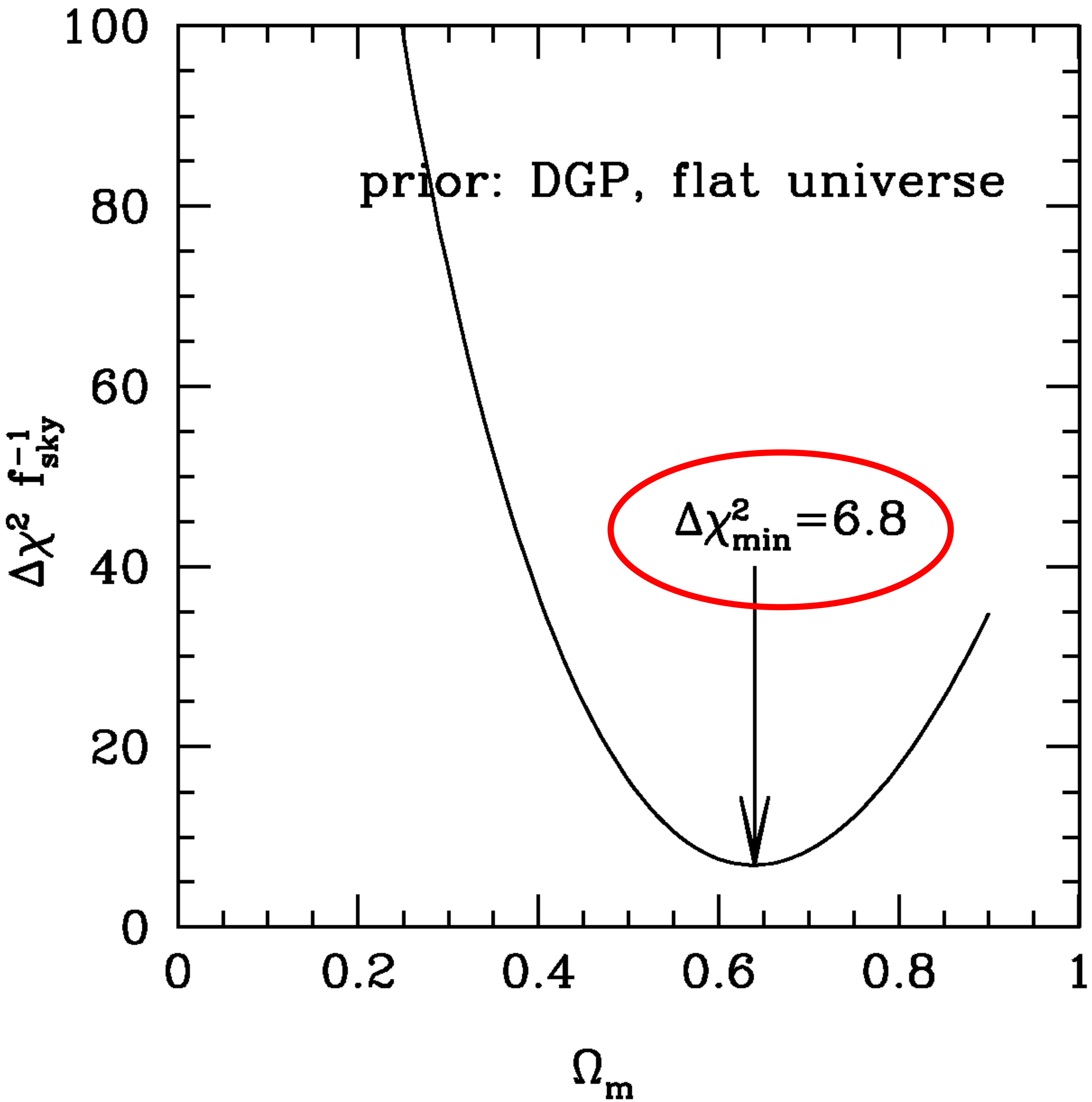


Fig. 3.— Distinguishing DGP from the fiducial ΛCDM cosmology. Since at high redshifts gravitational potential decays much faster in DGP than in ΛCDM cosmology, DGP can be distinguished from ΛCDM at $> 2.5\sigma$ confidence level.

- For details of DGP, refer to Y. Song's talk

Zhang 2006a

Testing gravity against z~3 ISW effect

- In general relativity, linear density growth factor D is scale *independent*, at sub-horizon.
  - The energy conservation:

$$\boxed{\dot{\delta} + \theta = 0}$$

  - The momentum conservation:

$$\boxed{\dot{\theta} + 2H\theta + \frac{1}{a^2}\nabla^2\psi = 0}$$

  - The Poisson equation

$$\boxed{\nabla^2\psi = 3H_0^2\Omega_0 a^{-1}\delta}$$

- Modifications to general relativity in general, causes D to be scale dependent.
  - For example, changing the Possion equation, D will be in general scale dependent.

- This feature of scale dependent D can be in principle be detected through
  - weak lensing (e.g. White & Kochanek 2001, Stabenau & Jain 2006)
  - galaxy clustering (e.g. Shirata et al. 2005, Sealfon et al. 2005)
  - the late time ISW effect

- It can also cause an observable ISW effect at z~3!

An example of modified gravity

- $f(R)$ gravity with action

$$L = \int (R + f(R)) \sqrt{g} d^4x + L_{\text{matter}}$$

- field equation

$$(1 + f_R) R_{uv} - \frac{g_{uv}}{2}(R + f - 2\Box f_R) - f_{R;u;v} = 8\pi G T_{uv}$$

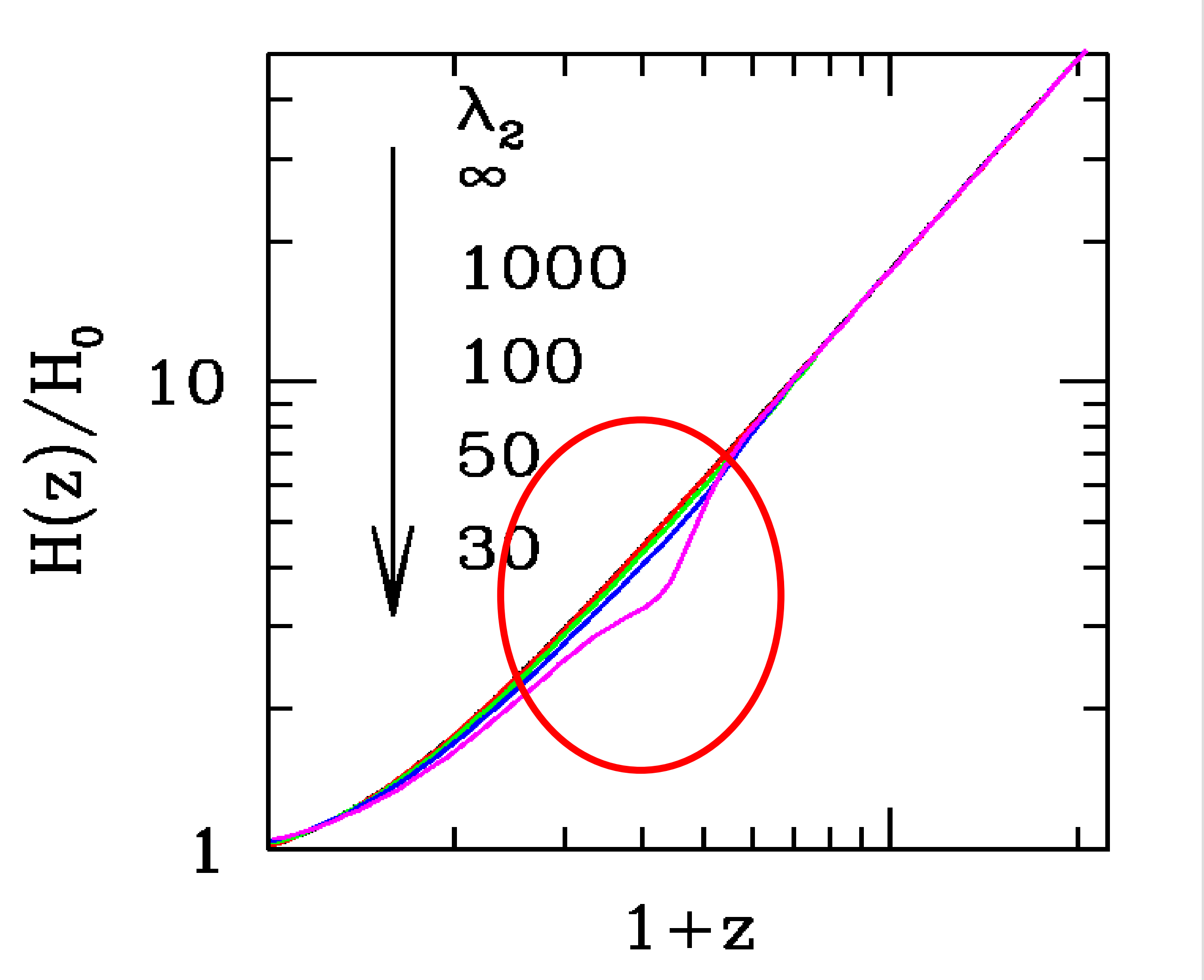
where $f_R \equiv df/dR$.

Fourth order derivative
in the metric

To accalarate the universe while passing the solar system tests, we choose a special class of $f(R)$ gravity.

- $f(R) = -\lambda_1 H_0^2 \exp(-R/\lambda_2 H_0^2)$, where $\lambda_{1,2}$ are two positive dimensionless constants and H_0 is the Hubble constant at present.
- To mimic a ΛCDM universe, $\lambda_1 \sim 1$ is required.
- To pass the solar system tests, $\lambda_2 \ll 10^6$.

- Zero order: the H-z relation.
  - $$H^2 = H_0^2\Omega_0 a^{-3} + [-\frac{f}{6} + \frac{\ddot{a}}{a} f_R - H\dot{f}_R]$$
  - For $\lambda_2 \geq 100$, the $H(z)$-z relation is almost identical to the corresponding ΛCDM cosmology.



- The Newtonian gauge

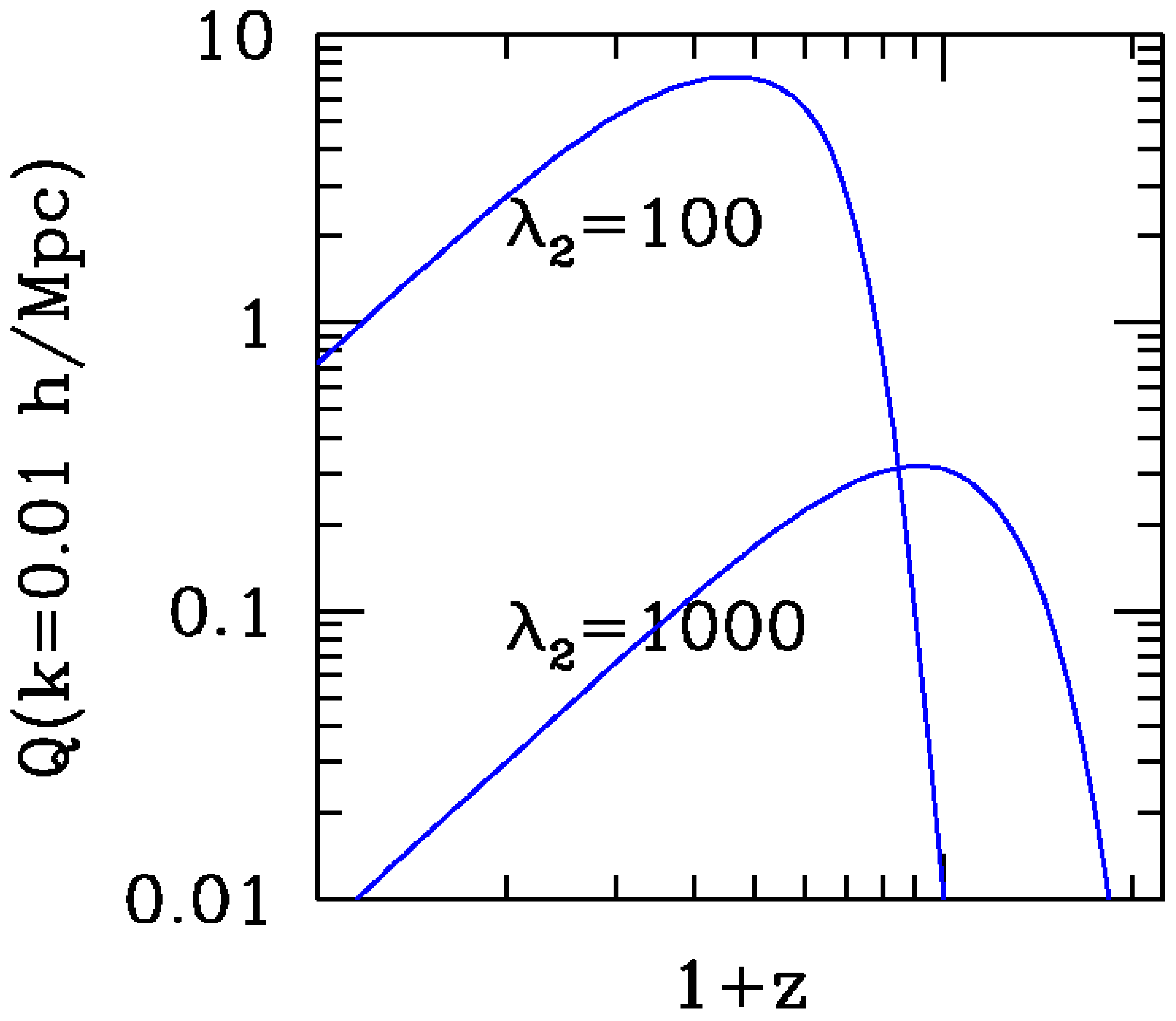
$$ds^2 = -(1+2\psi)dt^2 + a^2(1+2\phi)\sum_{i=1}^{3}(dx^i)^2$$

- Four perturbation variables ϕ, ψ, the matter over-density δ and the (comoving) peculiar velocity convergence θ.

- $\phi \neq -\psi$ due to non-vanishing $f_{R;\mu;\nu}$
- Taking the sub-horizon limit ($k \gtrsim aH$)

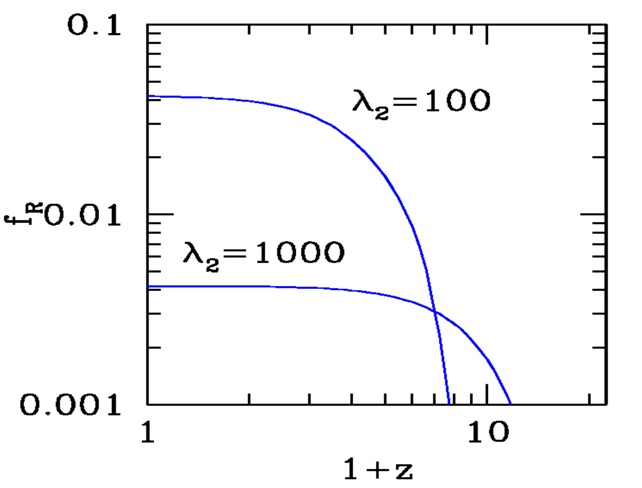
$$\phi + \psi = \frac{f_{RR}c^2}{1+f_R}\frac{2}{a^2}(\nabla^2\psi + 2\nabla^2\phi)$$

- In Fourier space. $\psi = -\phi(1-2Q)/(1-Q)$, where $Q(k,a) \equiv -2f_{RR}c^2k^2/(1+f_R)a^2$ and $f_{RR} \equiv d^2f/dR^2$.
- this scale dependent ψ-ϕ relation has profound effect on the LSS


10
1
0.1
0.01
Q(k=0.01 h/Mpc)
λ2=100
λ2=1000
1+z

- The new Poisson equation

$$\nabla^2(\phi - \psi) = -\frac{3H_0^2\Omega_0}{1+f_R}a^{-1}\delta$$



- The energy-momentum tensor is still conserved

$$\dot{\delta} + \theta = 0\ , \quad \dot{\theta} + 2H\theta + \frac{1}{a^2}\nabla^2\psi = 0$$

- The evolution of matter overdensity

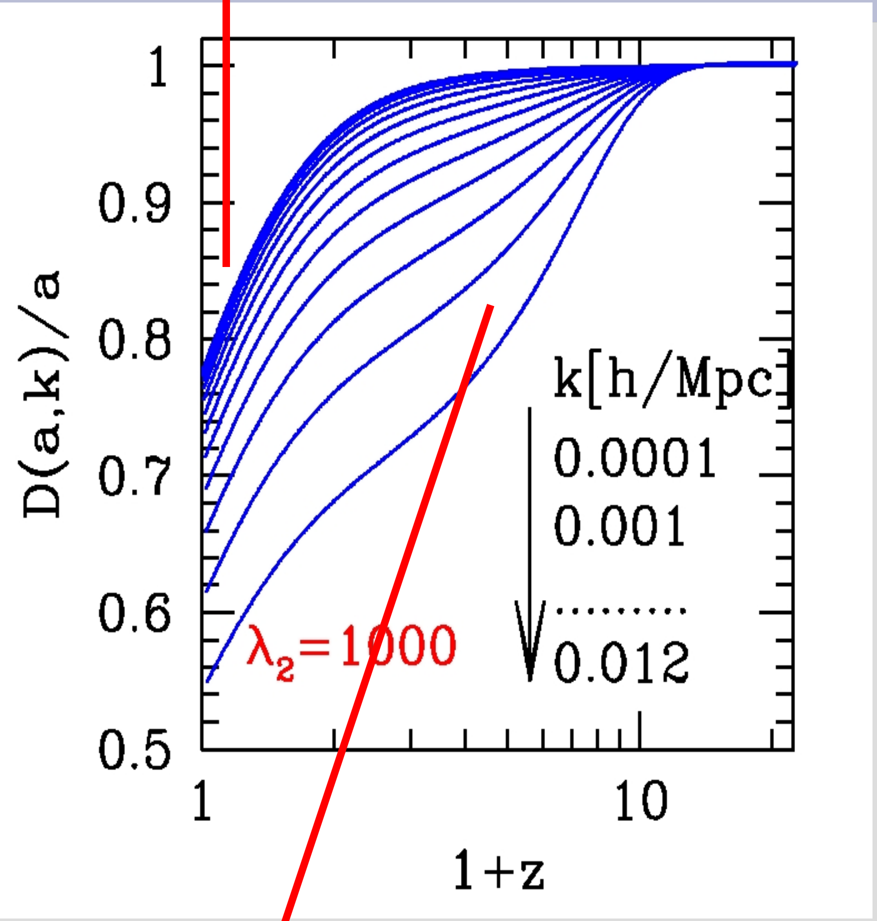
$$\delta'' + \delta'\left(\frac{3}{a} + \frac{H'}{H}\right) - \frac{\delta}{a^2}\frac{1-2Q}{2-3Q}\frac{3H_0^2\Omega_0}{a^3H^2(1+f_R)} = 0$$

where $' \equiv d/da$.

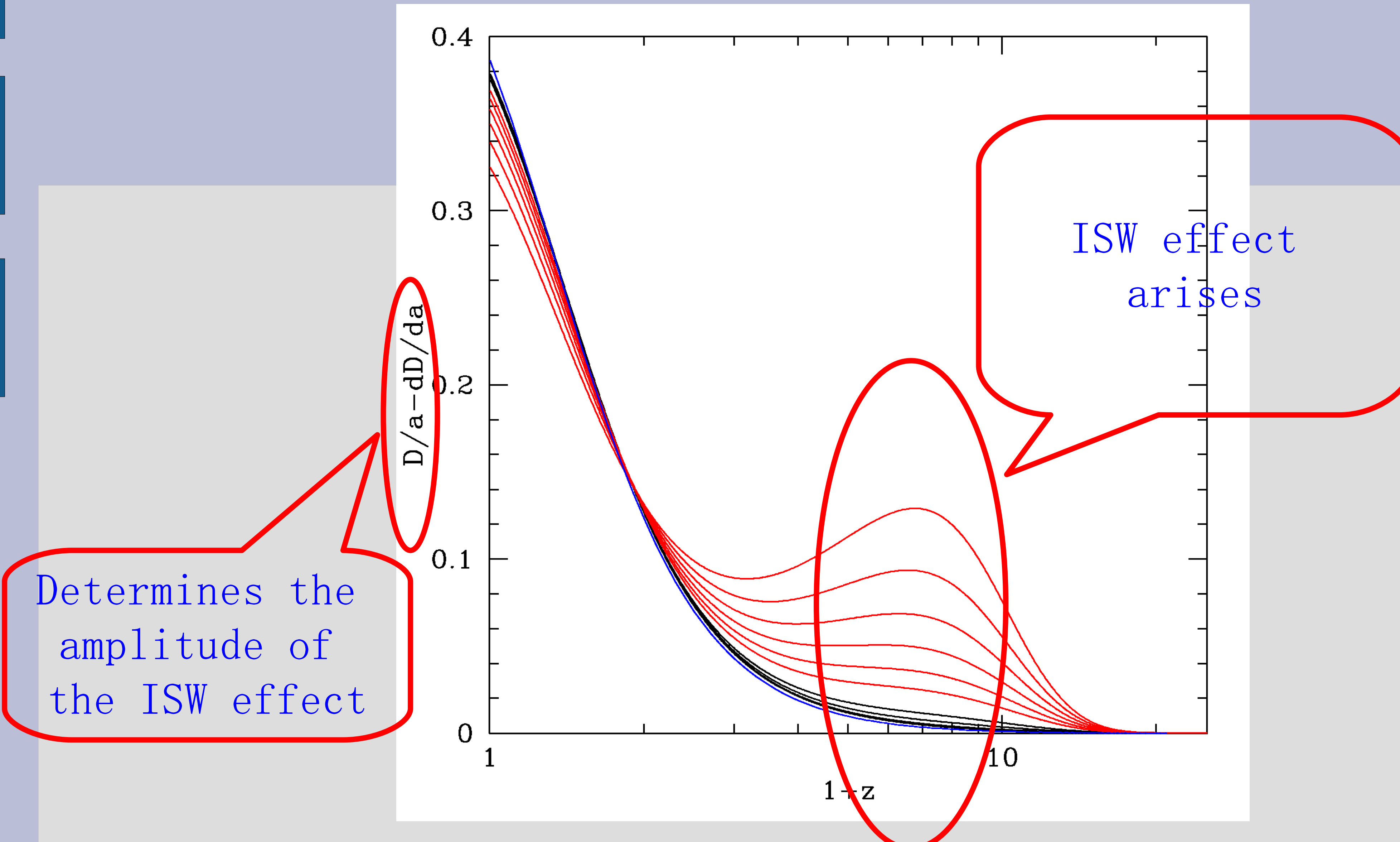
Scale dependence

Modified G

- we choose $\lambda_2 = 1000$
- At later time when $R \lesssim \lambda_2 H_0^2$, $Q \to 0$, the evolution of D approaches that of ΛCDM.

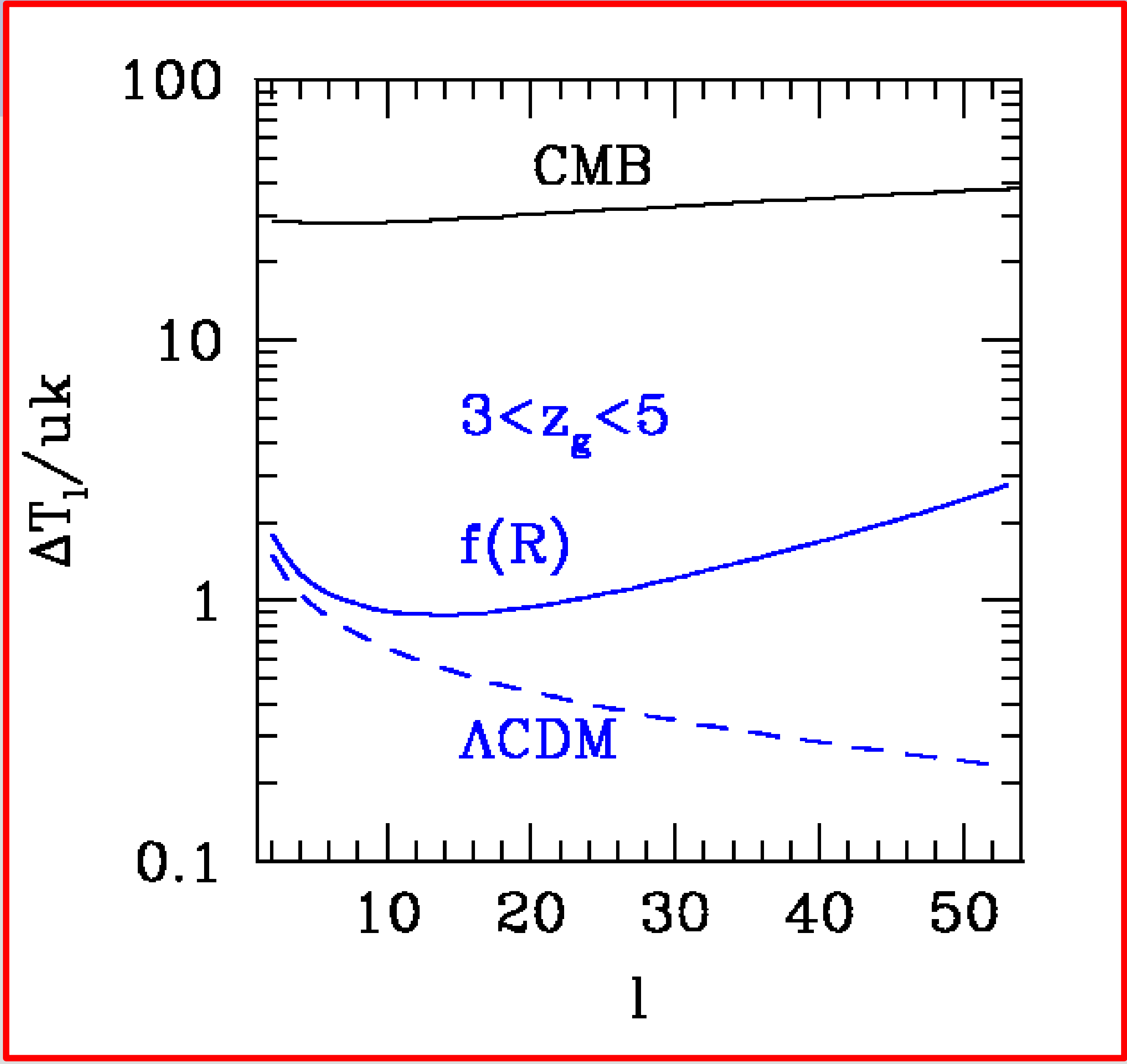


- At $z \gg 1$, $H \propto a^{-3/2}$, $D \propto a^{1-\eta}$ when $\eta \equiv 3Q/5(2-3Q) \ll 1$. Thus gravitational potential decays at high redshifts with rate $\propto a^{-\eta}$.

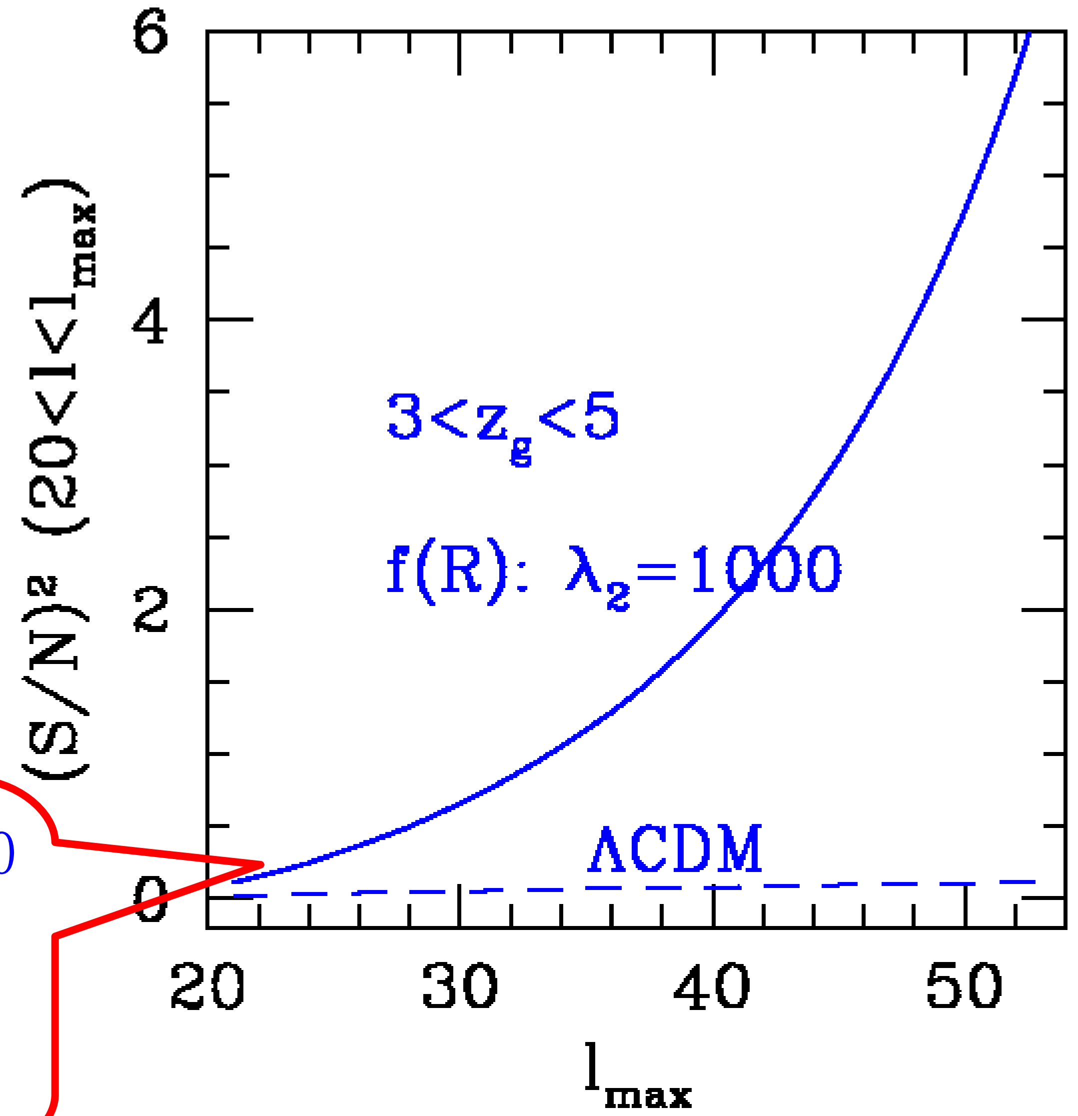


0.4
0.3
0.2
0.1
0
D/a-dD/da
1
10
1+z
ISW effect arises
Determines the amplitude of the ISW effect

The ISW effect in f(R) gravity



The detectability



Disregard l<20 to avoid possible confusions

Zhang 2006b

Summary

- The ISW effect is not only a powerful probe of dark energy, but also a powerful probe of modified gravity
-
- High z ISW effect contains important information of gravity
-
- Cautions: some dark energy models can in principle mimic modified gravities.