Impact of Non-Gaussian errors on CMB lensing and Dark Energy observables in the lensed CMB

Kendrick Smith
University of Chicago
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References: Smith, Hu and Kaplinghat, astro-ph/0402442
Smith, Kaplinghat and Hu, to appear

What information is in the primary (unlensed) CMB?

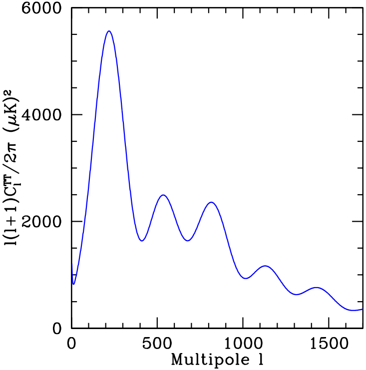
From shape of the power spectra, get strong constraints on:

$$\{\Omega_b h^2, \Omega_m h^2, n_s\}$$

Adding polarization at low multipoles, can separate:

$$\{\tau, \ln(\delta_\zeta)\}$$

(Without this information, $(\ln(\delta_\zeta) - \tau)$ is very well measured, but only this combination.)



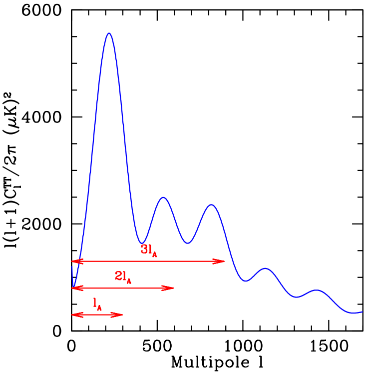
What information is in the primary (unlensed) CMB?

Finally, can measure angular scale of the acoustic peaks:

$$\ell_A = \pi \frac{D_*}{s_*} \quad \begin{array}{l} \leftarrow \text{Angular diameter distance to recombination} \\ \leftarrow \text{Sound horizon at recombination} \end{array}$$

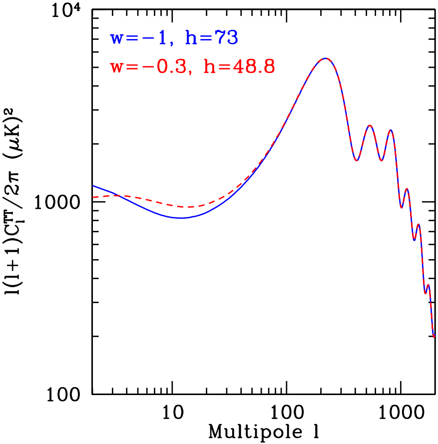
This can be interpreted (via the distance measurement D_*) as one constraint in a dark energy parameter space.

e.g. Hubble parameter h is well measured if $w = -1$ is assumed (WMAP3: $h = 0.73 \pm 0.03$) or prior on h constrains w



But, because there is only one observable, large degeneracies exist.

Angular diameter distance degeneracy (in unlensed CMB)



Models with $w = -1$ (and $h = 0.73$) and $w = -0.3$ (and $h = 48.8$) are nearly degenerate in the unlensed CMB

Gravitational lensing

Intevening matter between the surface of last scattering and an observer today lenses the CMB:

$$\widetilde{T}(\widehat{n}) = T(\widehat{n} + \nabla\phi(\widehat{n})),$$

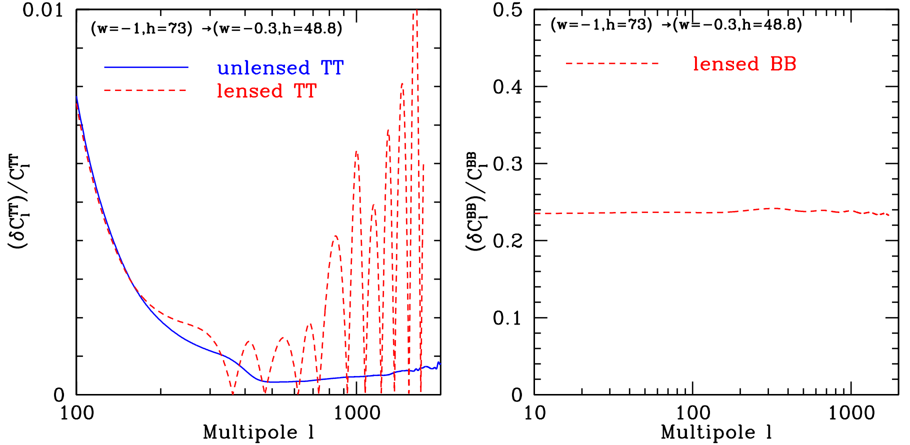
where ϕ is the projected potential, given by the line-of-sight integral

$$\phi(\widehat{n}) = -2\int dD\,\frac{(D_s - D)}{DD_s}\Psi(D\widehat{n}, D)$$

$\Rightarrow$ The CMB has some sensitivity to structure at $z \sim 1$

Lensing breaks the angular diameter distance degeneracy

Fractional difference between w=-1 and w=-0.3 models:



Scope of talk

Questions:

- What dark energy observables are contained in the lensed CMB? (complementing $\ell_A = \pi D_* / s*$)
- Are parameter constraints from the lensed CMB affected by non-Gaussian statistics?

Caveats:

- Parameter forecasts computed in the Fisher matrix approximation
- Only parameter constraints from power spectra (i.e., two-point statistics) are considered. Higher-order statistics (such as lens reconstruction methods) may improve constraints.

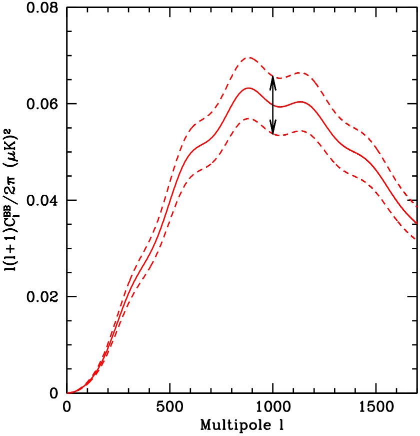
Why is the lensed CMB non-Gaussian?

$$
\begin{aligned}
\widetilde{T}(\widehat{n}) &= T(\widehat{n} + \nabla\phi(\widehat{n})) \\
&= T + (\nabla^a\phi)(\nabla_a T) + \frac{1}{2}(\nabla^a\phi)(\nabla^b\phi)(\nabla_{ab} T) + \cdots
\end{aligned}
$$

- Terms starting with the second are non-Gaussian (products of Gaussian fields)

Non-Gaussianity: intuitive argument

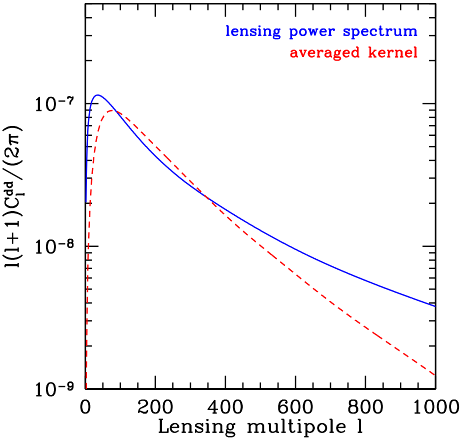
Consider the question: how well can the overall amplitude of the lensing B-modes be measured?



Cosmic variance limited, $\ell_{max} = 2000$: 0.07%

Non-Gaussianity: intuitive argument

Now ask: Which angular scales in the lensing potential contribute to the overall B-mode amplitude?

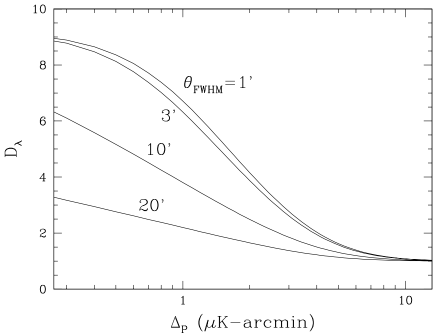


Sample variance limit: 0.20% (assuming ficticious direct measurement of ϕ)

Apparent contradiction

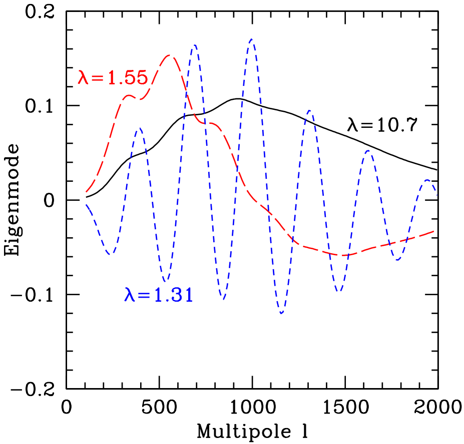
- Seem to get better constraint from measuring the lensed B-modes (0.07%) than measuring the unlensed CMB *and* the lensing potential ϕ (0.20%).

- However, the analysis has implicitly assumed Gaussian statistics for BB. When non-Gaussian contributions to the BB power spectrum covariance are included, the variance of the overall amplitude degrades (by the factor D_λ shown).



Non-Gaussianity: complete treatment

- Non-Gaussianity is always negligible in {TT,TE,EE}
- In BB, non-Gaussianity increases the variance of a few eigenmodes

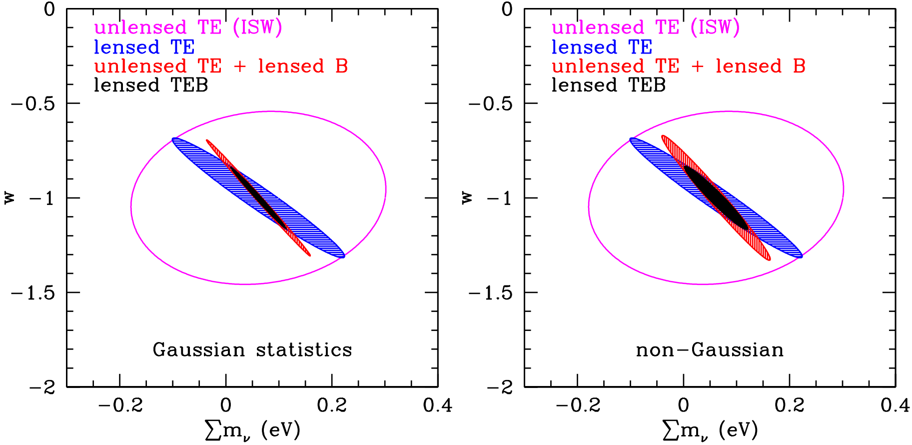


- Noise level has to be very good for non-Gaussianity to be important!

Parameter Estimation Example (fixed $\Omega_m h^2$, $\ln(\delta_\zeta)$)

Reference survey: 20 μK-arcmin, all-sky, zero beam
$+$ 1 μK-arcmin, $f_{sky} = 0.1$, zero beam

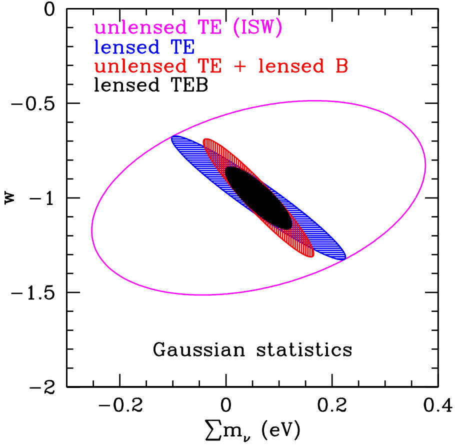
Gaussian vs non-Gaussian uncertainties on $(\Omega_\nu h^2, w)$:

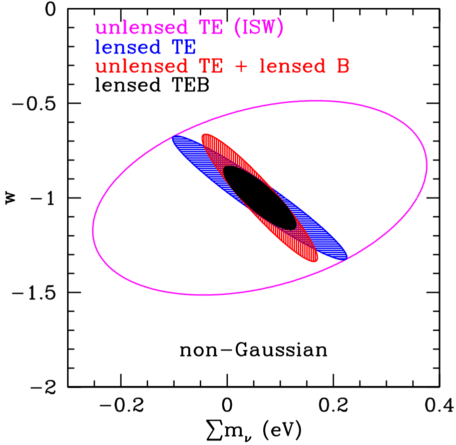


Parameter Estimation Example (fully marginalized)

Reference survey: 20 μK-arcmin, all-sky, zero beam
$+ 1$ μK-arcmin, $f_{sky} = 0.1$, zero beam

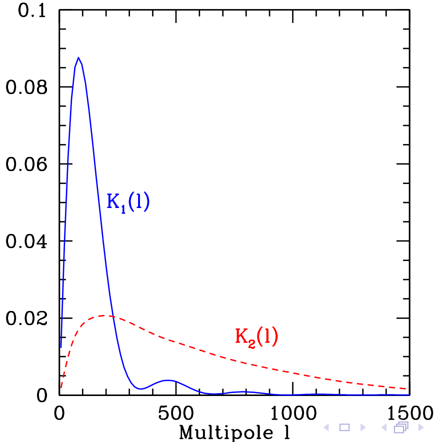
Gaussian vs non-Gaussian uncertainties on $(\Omega_\nu h^2, w)$:





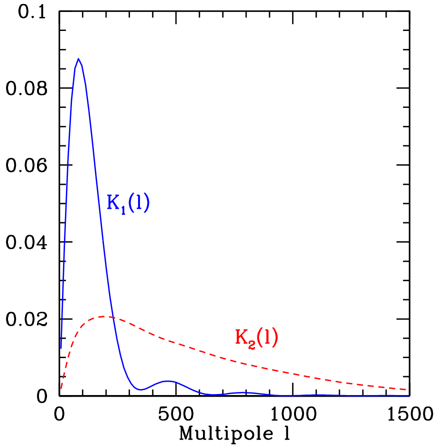
Which modes in $C_\ell^{\phi\phi}$ can CMB lensing constrain?

- Principal component analysis
- {TT,TE,EE} constrain one principal component $K_1(\ell)$ at $\ell \sim 100$
- BB constrains a distinct principal component $K_2(\ell)$ across a wide range of ℓ

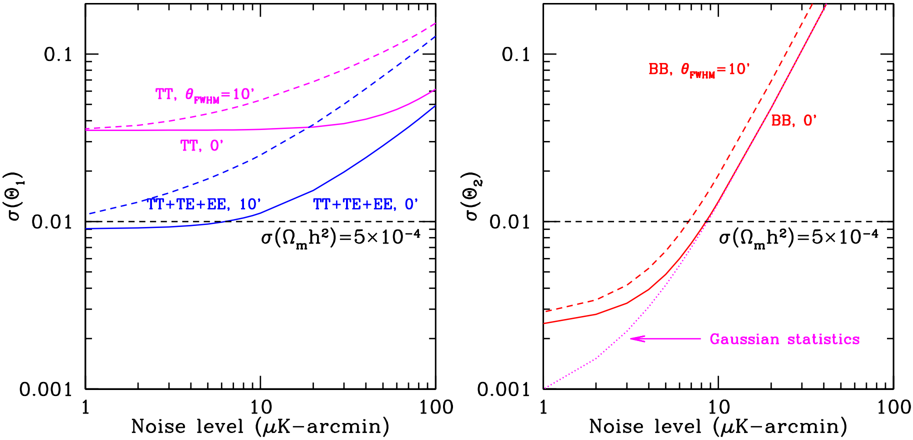


Dark energy observables defined

$$\Theta_1 = \sum_{\ell} C_\ell^{\phi\phi} K_1(\ell) \qquad \Theta_2 = \sum_{\ell} C_\ell^{\phi\phi} K_2(\ell)$$



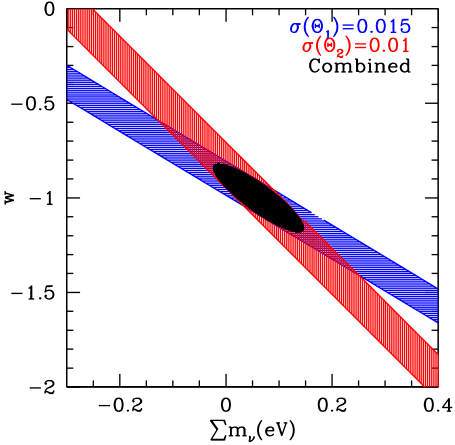
Dark energy observables: sensitivity

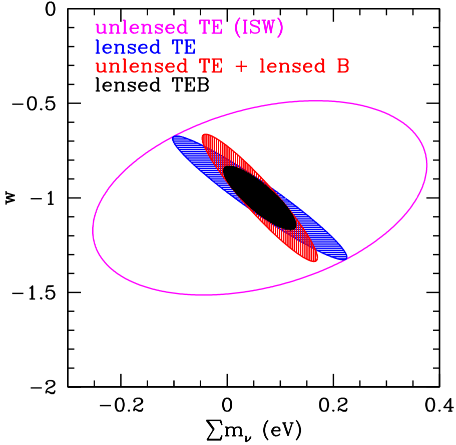


Dark energy observables: example

Reference survey: 20 μK-arcmin, all-sky, zero beam
+ 1 μK-arcmin, $f_{sky} = 0.1$, zero beam

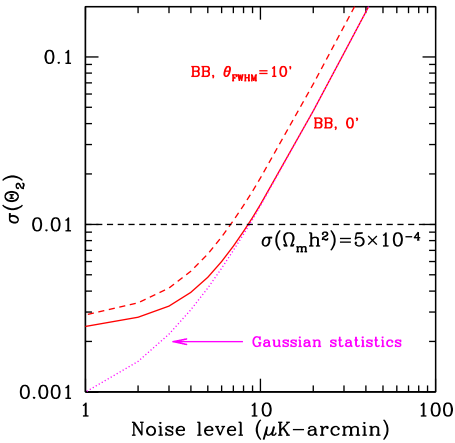
$\{\Theta_i\}$ picture vs complete Fisher calculation:

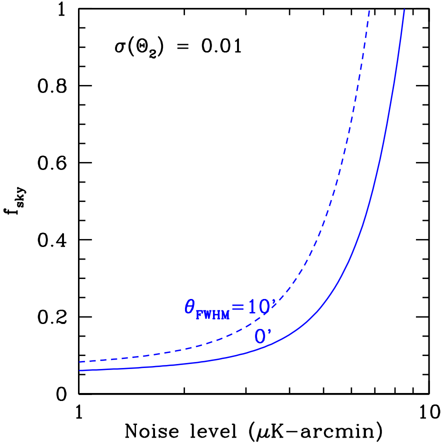




Application 1: existence of a B-mode sensitivity "floor"

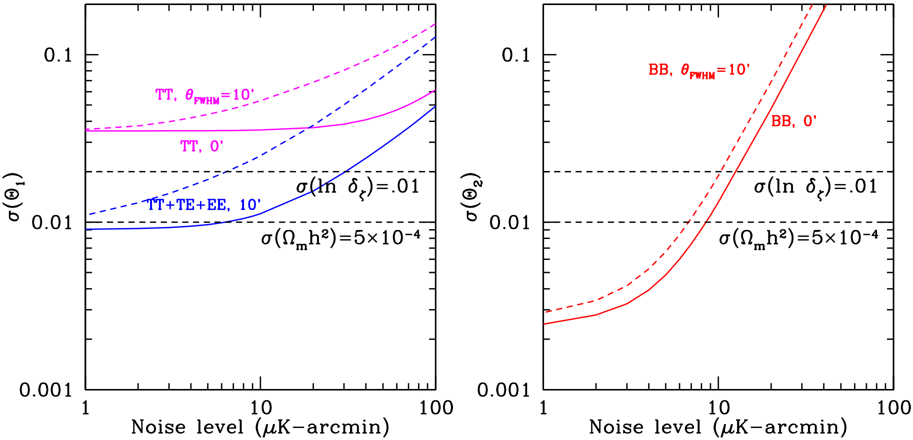
- Uncertainty in $\{\Omega_m h^2\}$ makes a ~ 0.01 contibution to $\sigma(\Theta_2)$
- Improving experimental sensitivity beyond this level will not improve dark energy constraints





Application 2: reionization history uncertainties

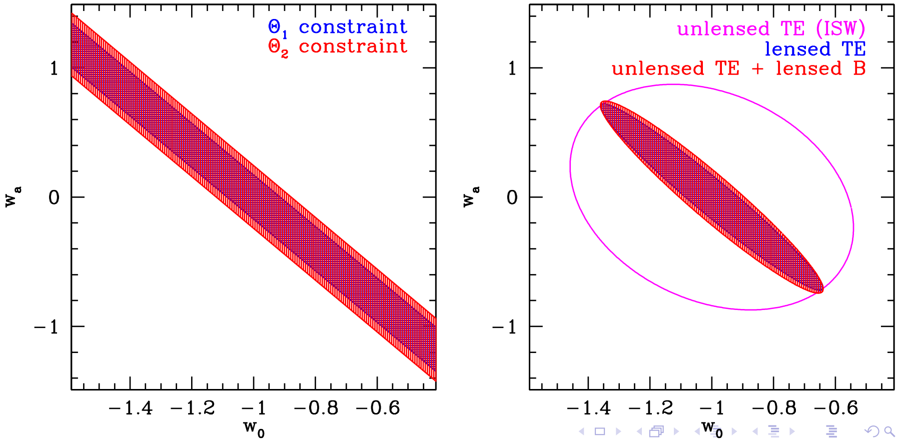
- Without assuming sharp reionization, $\sigma(\tau) \sim 0.01$ is best possible
- Uncertainties in Θ_i from $\ln(\delta_\zeta)$ (rather than $\Omega_m h^2$) would then be the limiting factor at high sensitivity



Application 3: constraints on (w_0, w_a)

Fixing neutrino mass, but allowing $w(a) = w_0 + (1-a)w_a$.
Reference survey: 20 μK-arcmin, all-sky, zero beam
$+ 1$ μK-arcmin, $f_{sky} = 0.1$, zero beam

$\{\Theta_i\}$ picture vs complete Fisher calculation:



Summary

- The unlensed CMB places excellent constraints on one dark energy observable $\ell_A = \pi D_* / s_*$ but is otherwise degenerate.
- Through lensing, the CMB is sensitive to density fluctuations at $z \sim 1$, which break the degeneracy.
- Lensed {T,E} can constrain a second observable Θ_1; lensed B can constrain a third observable Θ_2.
- Non-Gaussianity in lensed {TT,TE,EE} is always negligible.
- Non-Gaussianity in lensed BB degrades the overall amplitude uncertainty by a factor of ~ 10; this does not affect dark energy uncertainties after marginalizing $\Omega_m h^2$.
- There is a B-mode sensitivity "floor", beyond which dark energy constraints do not improve.
- The parameters w and w_a cannot be separated using the CMB alone.